Performance Analysis of Compact Heat Exchanger

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Abstract

This paper presents methodology for analysis of performance of compact heat exchanger (CHE). A deterministic quantitative model based on graph theoretical methodology has been developed for calculating the permanent function and numerical index of compact heat exchanger performance factors. The permanent function is used for evaluation and analysis of compact heat exchanger. The compact heat exchanger (CHE) performance factors are divided into subsystems and interactions between subsystems are represented into matrix form. This paper provides a general formula which can be applied to different compact heat exchangers (CHE).
Keywords: CHE, Matrix, Performance, Permanent Function

I. INTRODUCTION

A heat exchanger is a device to transfer heat from a hot fluid to cold fluid across an impermeable wall. Fundamental of heat exchanger principle is to facilitate an efficient heat flow from hot fluid to cold fluid. This heat flow is a direct function of the temperature difference between the two fluids, the area where heat is transferred, and the conductive/convective properties of the fluid and the flow state. This relation was formulated by Newton and called Newton's law of cooling, which is given in Equation

$$Q = h*A*T \qquad (1)$$

Where h is the heat transfer coefficient [W/m^2K], where fluid's conductive/convective properties and the flow state comes in the picture, A is the heat transfer area [m^2], and T is the temperature difference [K].Heat exchangers are one of the vital components in diverse engineering plants and systems. So the design and construction of heat exchangers is often vital for the proper functioning of such systems. It has been shown in [Barron, 1985] that the exchanger is below 86.9%. On the other hand in aircrafts and automobiles, for a given heat duty, the volume and weight of the heat exchangers should be as minimum as much as possible. Heat exchangers constitute the most important components of many industrial processes and equipment's covering a wide range of engineering applications. Increasing awareness for the effective utilization of energy resources,

Minimizing operating cost and maintenance free operation have led to the development of efficient heat exchangers like compact heat exchangers. These heat exchangers have a very high heat transfer surface area with respect to their volume and are associated with high heat transfer coefficients. Typically, the heat exchanger is called compact if the surface area density (β) i.e. heat transfer surface area per unit volume is greater than 700 m2/m3 in either one or more sides of two-stream or multi stream heat exchanger[R.K Shah, Heat Exchangers, Thermal Hydraulic 1980].

II. GRAPH THEORETIC APPROACH (GTA)

Graph theory is a logical and systematic approach useful for modeling and analyzing various kinds of systems and problems in numerous fields of science and technology [1-2]. GTA synthesizes the inter-relationship among different parameters or sub-systems parameters and provides a synthetic score for the entire system. It also takes care of directional relationship and inter-dependence among parameters.

Graph theoretic and matrix model consists of diagraph representation, matrix representation and permanent representation. The diagraph characterizes the visual representation of the elements (parameters) and their interdependence. The matrix converts the diagraph into mathematical form. The permanent function is a mathematical model that helps to determine index, which can be used for comparison.

A. Assumptions for Developing Graph Theoretic Model:

The proposed graph theoretic model for EDM systems is based on some assumptions as listed below:-

1) The structure of the system can be compared quantitatively with its performance.
2) The interactions as well as the subsystems discussed are assumed for a general CHE system. The subsystems must be identified separately for applying the model to any specific CHE system.

3) Variable permanent matrix is capable of storing complete information related to a real life situation of a typical CHE system as all its elements are variables and functions of characterizing attributes representing subsystem and interconnections. These attributes if identified comprehensively, the matrix represents the CHE system completely.
4) Permanent function of the variable permanent matrix characterizes uniquely the CHE system form the point view of the structure.
5) Performance of the CHE system depends on individual performance of subsystems, sub-systems and their components along with interactions between them.
6) Permanent function values of sub-subsystems are used in permanent matrices of subsystems. Permanent function values of subsystems are used to calculate permanent functions of CHE system.
7) The experts may assign correct and representative score to the elements of the CHE system at the lowest level in a particular dimension of performance.

III. IDENTIFICATION OF CHE PERFORMANCE FACTORS

The factors identified in literature article are categorized into four categories. Grouping is done because number of factors identified is large and due to which calculation of matrices becomes a very difficult task. These factors are categorized as below.

A. Heat Transfer Coefficient (B1):

Accurate and reliable dimensionless heat transfer and pressure drop characteristics are a key input for designing or analyzing a plate fin heat exchanger. For single-phase flow, the heat transfer coefficient is generally expressed in terms of the Colburn correlation [Kern and Kraus, 1972]

$$h = j\ Cp\ G\ (Pr)^{-2/3} \qquad (2)$$

Where j called as colburn factor separates the effects of the fluid properties on the heat transfer coefficient and permits correlations as a function of the Reynolds number (Re). While the j data are expressed as functions of Prandtl number (Pr) and Re, temperature does not appear directly in the expression. Temperature has the role in determining thermo-physical properties such as density, viscosity, specific heat. Therefore, it is generally recognized that j data determined at one temperature / pressure level and expressed in dimensionless form are directly usable at another temperature / pressure level.

B. Pressure Drop (B2):

The plate fin heat exchangers are mainly used for gas to gas heat transfer applications and most of the gases are low density gases, so the pumping power requirement in a gas-to-gas heat exchanger is high as compared to that in a liquid-to-liquid heat exchanger. This fact makes it mandatory to have an accurate estimation of friction characteristics of the heat exchanger surfaces in gas application. The friction factor is defined on the basis of an equivalent shear force in the flow direction per unit friction area. This shear force can be either viscous shear (skin friction) or pressure force (form drag) or a combination of both. The pressure drop (ΔP) for internal flow through ducts can be calculated by equation (3)

$$\Delta P = \frac{fL\rho V^2}{2D} \qquad (3)$$

It can be seen that temperature does not appear directly in the expression of friction factor also. Therefore, the f data determined at one temperature / pressure level are directly usable at other temperature / pressure level. But it is seen that j and f are strong functions of fin geometries like fin height, fin spacing, fin thickness etc. Because fins are available in varied shapes, it becomes necessary to test each configuration individually to determine the heat transfer and flow friction characteristics for specific surface. For a given fin geometry, in general, increase in heat transfer performance is associated with increase in flow friction and vice versa. Customarily the ratio of j/f is taken as a measure of the goodness of the fin surface. Though the preferred fin geometry would have high heat transfer coefficient without correspondingly increased pressure penalty, the selection of particular fin geometry mainly depends on the process requirement; one can sacrifice either of heat transfer or pressure loss at the cost of other. The Reynolds number is kept low because the hydraulic diameter of the flow passages is generally small due to closely spaced fins and in such conditions operation with low density gases leads to excessive pressure drop unless the gas velocity in the flow passage is kept low.

C. Fin Dimensions (B3):

A fin is a surface that extends from an object to increase the rate of heat transfer to or from the environment by increasing convection. The amount of conduction, convection, or radiation of an object determines the amount of heat it transfers. Increasing the temperature difference between the object and the environment, increasing the convection heat transfer coefficient, or increasing the surface area of the object increases the heat transfer. A plate fin heat exchanger with offset strip fin geometry described by the following parameters:

1) Fin spacing (s), excluding the fin thickness,
2) Fin height (h), excluding the fin thickness,
3) Fin thickness (t) and,

4) Fin strip length (l or L_f)

The lateral fin offset is generally the same and equal to half the fin spacing (including fin thickness). Figure 3.3 shows a schematic view of the rectangular offset strip fin surface and defines the basic geometric parameters.

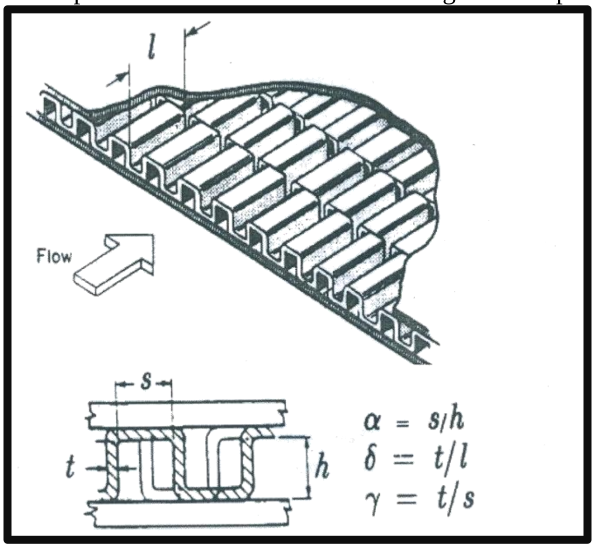

Fig. 1: Geometry of Typical Offset Strip Fin Surface

IV. Graph Theoretic Modeling of an EDM System

A graph theory based modeling of a CHE system is proposed. A graph theory based mathematical model is derived for representing the physical structure of CHE system along with interactions. Four sub systems of CHE system are discussed above and interactions among them are represented using a digraph as shown in fig. 4.1. In general graph is represented mathematically by G = {V, E}; where V represents vertices of the graph and E represents the edges connecting between those vertices. In the CHE system, let the vertices correspond to subsystems (Vi) and the edges (eij) correspond to interaction of its two sub systems. In fig. 4.1 components of CHE system are represented by the vertices of the graph and the interaction of the subsystems are represented by the edges of the graph. Undirected graph are used where directional property is not significant as shown in fig. 4.1. This means that the influence of ith vertex on jth vertex is equal to the influence of jth vertex on ith vertex. For directed graph i.e .fig. 4.1 shows system diagraph of CHE system.

The diagraph is nothing but a directed graph. The directed graph use pointed or directional edges to connect various variables. The digraph is the visual representation of the elements (factors) and their interdependence. It is easy to predict the relation between variables by visualization of diagraph. The second element is matrix representation. As diagraph is only for visualization and it cannot be convert into mathematical form. The matrix converts the digraph into mathematical form. Therefore matrix representation is very important. The permanent function is a mathematical model that helps to determine index which is helpful for comparison. The permanent function gives a single numerical value that represents the impact of factors.

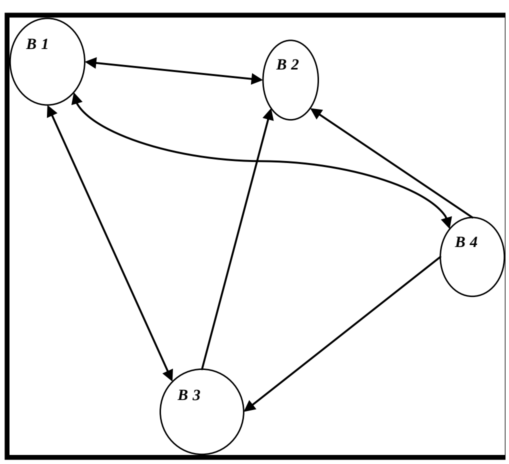

Fig 2: Diagraph Representation

V. MATRIX REPRESENTATION FOR THE EDM SYSTEM GRAPH

To quantify sub systems and interactions in EDM system graph an equal mathematical representation known as Matrix is used. In general the graph is represented by a matrix called adjacency matrix. Since adjacency matrix represents the physical structure of an EDM system, it is termed as EDM System Structure Matrix.

A. EDM System Matrix (EDMSM):

EDM System matrix is a square matrix, having rows and columns correspond to respective sub system. This matrix is a binary square matrix having (0, 1) as elements. EDM system of N sub systems is represented by a structure matrix of size N x N. The system structure matrix of the EDM system graph with four nodes will be a
Four order binary (0, 1) square matrix, A = [eij] such that:

$$eij = \begin{cases} = 1, \text{ if subsystem i is connected to subsystem.} \\ =0, \text{ otherwise.} \end{cases}$$

bii = 0; as a factor, is not interacting with itself.

This CHE is a square and non symmetric matrix.
The CHE factor matrix representing the diagraph of categories can be written as below:

$$A = \begin{array}{c} \begin{matrix} B1 & B2 & B3 & B4 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

The interdependency of CHE factors is shown by non diagonal elements with value 0 or 1. "1" means there is a directed edge between two factors and "0" means there is no such edge. The diagonal elements are 0 because the effect/inheritance of CHE factors is not taken into consideration. For this, another matrix known as characteristic matrix of CHE factors is used.

B. CHE Factors Characteristic Matrix (CHECM):

The characteristic matrix is generally used in mathematics and here it is used to characterize the CHE factors. Let us consider I is an identity matrix and B is a variable which represent CHE factors. The characteristic matrix for CHE factors is shown as below:

$$C = BI - A$$

$$B = \begin{array}{c} \begin{matrix} B1 & B2 & B3 & B4 \end{matrix} \\ \begin{bmatrix} B & 0 & -1 & -1 \\ -1 & B & -1 & 0 \\ -1 & 0 & B & 0 \\ 0 & -1 & -1 & B \end{bmatrix} \end{array}$$

In above matrix all diagonal elements have same values; it means all CHE factors were assigned the same value which is not true in practice, since all CHE factors have different values depending on various parameters affecting them. Moreover interdependencies were assigned values of 0 and 1 depending on whether it is there or not. To consider this, another matrix, the variable characteristic matrix of CHE factors, is taken into consideration.

C. CHE Factors Variable Characteristic Matrix (CHEVCM):

The Variable Characteristic matrix of CHE factors (VCM) considers the effect of different CHE barriers and their interactions. The digraph of categories is used for defining VCM. Let us consider a matrix P with non diagonal elements bij's representing interactions between CHE factors, instead of 1 (as shown in matrix A). Let us consider another matrix Q is taken with diagonal elements Bi, i = 1, 2, 3, 4, 5 where the Bi represents the effect of various barriers, i.e. instead of B only (as shown in matrix C). Using matrices P and Q, VCM can be expressed as.

$$VCM = Q - P$$

$$VCM = \begin{array}{c} \begin{matrix} B1 & B2 & B3 & B4 \end{matrix} \\ \begin{bmatrix} B1 & 0 & -b13 & -b14 \\ -b21 & B2 & -b23 & 0 \\ -b31 & 0 & B3 & 0 \\ 0 & -b42 & -b43 & B4 \end{bmatrix} \end{array}$$

The determinant of this matrix is very important and called variable characteristic CHE factor multinomial. It represents the CHE environment in the system by taking into consideration effect of factors and their interactions. The determinant of the matrix VCM contains positive and negative terms and hence some terms will lost due to addition and subtraction during the calculation of determinant. It implies complete information will not obtain due to cancelation of some terms. For this variable permanent matrix of CHE factors is defined

D. CHE Factors Variable Permanent Matrix (CHEVPM):

The using two matrices P and Q VPM can be expressed as:

$$VPM = P + Q$$

The diagonal elements of VPM matrix represent the effect of four critical CHE factors in implementation of CHE. The non diagonal elements represent the interdependency of each factor in the matrix.

$$\begin{matrix} & B1 & B2 & B3 & B4 \end{matrix}$$

$$VPM = \begin{bmatrix} B1 & 0 & b13 & 0 \\ b21 & B2 & b23 & b24 \\ b31 & b32 & B3 & b34 \\ b41 & b42 & b43 & B4 \end{bmatrix}$$

VI. PERMANENT FUNCTION REPRESENTATION

Diagraph depends upon position of nodes and can change by changing the nodes. Similarly matrix also depends upon labeling of nodes. Therefore both diagraph and matrix are not unique. For making a unique representation, a permanent function of matrix VCM is defined. Generally it is used in mathematics. It is obtained in the same fashion as determinant. The calculation of permanent function results in a polynomial which is very significant. It has complete information about CHE factors and it is called variable permanent function of CHE factors. In general form variable permanent function (VPF) can be expressed as:

$$VPF = \prod_1^4 Bi + \sum_i \sum_j \sum_k \sum_l (bijbji) BkBl + \sum_i \sum_j \sum_k \sum_l (bijbjkbkl + blkbkjbji) Bl + \left\{ \sum_i \sum_j \sum_k \sum_l (bijbji)(bklblk) + \sum_i \sum_j \sum_k \sum_l (bijbjkbklbli + bilblkbkjbji \right\}$$

The VPF is a mathematical relation in symbolic form and it ensures computation of the CHE factors. It has complete information about CHE factors as it considers the presence of all factors and their interdependencies. The above expression contains 4! terms and these terms are arranged in n + 1 grouping, where n is the number of elements (factors). The value of n is equal to four in the present case.

VII. CHE PERFORMANCE ANALYSIS

The methodology described in this chapter for complete performance analysis of CHE is summarized in step-by-step manner as below:

Select the desired EDM system.

1) Study the system and its subsystems (heat transfer coefficient, pressure drop, fin dimensions, friction factor), and also their interactions.
2) Develop a system graph of the CHE system with subsystems as vertices and edges for interaction between the vertices.
3) Using the graph in step 2, develop matrix similar to CHE system variable permanent matrix.
4) Calculate system permanent function.
5) The numerical index of an CHE system would be obtained by substituting the numerical values of subsystems and their interactions.
6) Different factors of CHE can be compared on the basis of permanent numerical index thus obtained. Necessary improvements and developments can be done on the basis of this approach.
7) From the above methodology the critical performance factor of CHE is to be evaluated.

VIII. CONCLUSION

1) This study presents a methodology to build the interactions between various subsystems of CHE.
2) This work identifies four subsystems (heat transfer coefficient, pressure drop, fin dimensions, friction factor) which parameterize the CHE.
3) This methodology consists of the CHE system structure digraph, the EDM system matrix, and EDM system permanent function. The system digraph is the mathematical representation of the structural characteristics and their interactions, useful for analysis. The CHE system matrix converts graph into mathematical form. This matrix representation is powerful tool for storage and retrieval of CHE systems in computer database and for calculation of permanent function.
4) This study presents the system permanent function which is a mathematical model characterizing the structure of the CHE system and also helps to determine the CHE system numerical index. This numerical index is used for analysis of CHE.
5) This work presents a framework for developing various quantitative performance indices of CHE in various dimensions of performance i.e. effectiveness and NTU.
6) The proposed approach is useful for product design engineer, researcher, and industries at conceptual stage. This methodology is useful while taking decision for selecting CHE from different types available for given application. This

approach can be used along with Cause and effect analysis and SWOT (Strength-Weakness- Opportunity-Threats) analysis on CHE system. It is further used for developing maintenance strategies.

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