# Denoising of Computed Tomography Images using Wavelet Transform

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## Abstract

Image denoising is one of the most significant tasks in image processing, analysis and image processing applications. Medical Imaging is one among the emerging application areas where the image denoising plays a vital role. In medical imaging, the additional techniques and systems introduce noises and artifacts in the medical image that leads to poor quality image. In this moment, image denoising is an essential pre-requisite, specially in Computed Tomography, which is an important and most common method in medical imaging. The significance of the denoising is mainly due to that the effectiveness of clinical diagnosis using CT image depends upon the quality of the image. In this work, we propose an efficient noise reduction technique for CT images using wavelet transformation and thresholding. The technique removes Additive noise from the CT images as well as it enhances the quality of the images.
**Keywords: Continuous Wavelet Transform, Digital Imaging And Communications In Medicine, Discrete Wavelet Transform.**

## I. INTRODUCTION

Digital signal processing (DSP) describes the science that tries to evaluate, generate and manipulate measured real world signals with the help of a digital computer. signals can be anything that is a collection of numbers, or measurements and the most commonly used signals include images, audio (such as digitally recorded speech and music) and medical and seismic data. The Fourier transform (FT) is probably the most popular transform used to obtain the frequency spectrum of a signal.

Noise removal or de-noising is an important task in image processing. Image enhancement is a collection of techniques that improve the quality of the given image that is making certain features of the image easier to see or reducing the noise. In general, the results of the noise removal have a strong influence on the quality of the image processing techniques.

Noise generated by electronic components in instrumentation is a common type of random signal that is present in much biomedical data even though contemporary electronic design minimizes this noise. Often those components of a signal which are not understood are classified as noise. The ultimate base for deciding what constitutes noise should be derived from considerations about the experimental or clinical measurements and the source of a signal. Ideally when a prior knowledge for judging whether certain components of a signal represent the desired measurement or not is known then the signal processing method is chosen to enhance the desired signal and reduce undesired signal components. In some cases this information may not be known and it may be necessary to examine the results of the signal processing steps to assess whether the output signal exhibits some apparent separation into desired and noise components.

The field of imaging provides many examples of both biomedical images and biomedical image processing. Computed Tomography (CT) image is excellent for showing abnormalities of the brain such as: stroke, hemorrhage, tumor, multiple sclerosis or lesions. In the CT basic signals are currents induced in a coil caused by the movement of molecular dipoles as the molecules resume a condition of random orientation after having been aligned by the imposed magnetic field. Signal processing is needed to detect and decode them, which is done in terms of the spatial locations of the dipoles (which is related to the type of tissue in which they are located). Much of the joint signal processing is based on Fourier transform. Since CT utilizes two-dimensional Fourier transforms the basic concepts are the same

### *A. CT Imaging*

Computed Tomography (CT) image scan is an imaging technique used primarily in medical field to produce high quality images of the soft tissues of the human body. Using brain images acquired by CT often allows physicians and engineers to analyze the brain without the need for invasive surgery. Other types of imaging mdalities which exist include ultrasound imaging, X-ray imaging, Magnetic resonance imaging (MRI).CT combines X-ray machine with an advanced computer system and radio waves to produce correct, detailed pictures of organs and tissues in order to diagnose a variety of medical conditions. There are two types of CT exams namely the high-field CT and low-field open CT. The difference is in that high-field CT produces a highest quality image in the shortest time allowing a most accurate diagnosis to be made. Since CT can give high quality clear pictures of soft-tissue structures near and around bones, it is the most sensitive exam for brain, spinal and joint problems. CT is widely used to diagnose sports related injuries, especially those affecting the knee, brain. The images allow the physician to see even very small tears and injuries to ligaments and muscles

## II. BASICS OF WAVELET

A 'wavelet' is a small wave which has its energy concentrated in time. It has an oscillating wave like characteristic but also has the ability to allow simultaneous time and frequency analysis and it is a suitable tool for transient, non-stationary or time-varying phenomena [11,12].

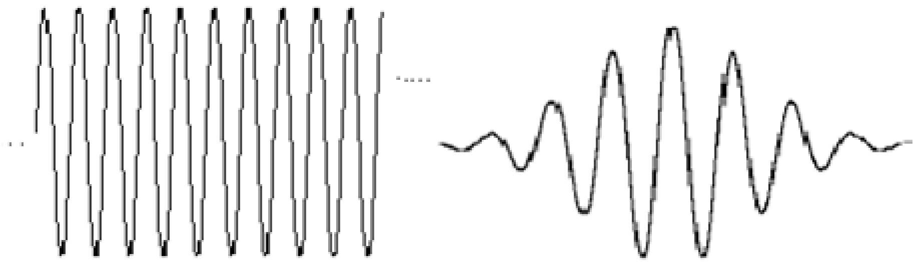

Fig. 1.1: (A) Wave (B) Wavelet

### A. *Wavelet History*

Wavelet theory has been developed as a unifying framework only recently, although similar ideas and constructions took place as early as the beginning of the century. The idea of looking at a signal at various scales and analyzing it with various resolutions has in fact emerged independently in many different fields of mathematics, physics and engineering. In mid-eighties, researchers of the 'French school' built strong mathematical foundation around the subject and named their work wavelets.

### B. *Wavelet Characteristic*

Waves are smooth, predictable and everlasting, whereas wavelets are of limited duration, erratic and may be asymmetric. Waves are used as deterministic basic functions in Fourier analysis for the expansion of functions (signals), which are time-invariant, or stationary. The essential characteristic of wavelets is that they can serve as deterministic or nondeterministic basis for generation and analysis of the most natural signals to provide better time-frequency representation, which is not possible with waves using ordinary Fourier analysis

### C. *Wavelet Analysis*

The wavelet analysis procedure is to adopt a wavelet prototype function, called an 'analyzing wavelet' or 'mother wavelet'. Temporal examination is performed with a contracted, high frequency form of the prototype wavelet, while frequency analysis is performed with a dilated, low frequency form of the same wavelet. Mathematical formation of signal expansion using wavelet gives Wavelet Transform (WT) pair, which is analogous to the Fourier Transform (FT) pair. Discrete-time and discrete-parameter version of WT is termed as Discrete Wavelet Transform (DWT). DWT can be viewed in a similar framework of Discrete Fourier Transform (DFT) with its efficient implementation through fast filter bank algorithms similar to Fast Fourier Transform (FFT) algorithms [13].

## III. IMAGE DE-NOISING

The reduction of noise present in images is an important aspect of image processing. Denoising is a procedure to recover a signal that has been corrupted by noise. After discrete wavelet decomposition the resulting coefficients can be modified to eliminate undesirable signal components. To implement wavelet thresholding a wavelet shrinkage method for de-noising the image has been verified. The algorithm to be used is summarized in Algorithm 1 and it consists of the following steps.

### A. *Algorithm 1: Wavelet image de-noising*

- Choice of a wavelet and number of levels or scales for the decomposition. Estimation of a threshold.
- Choice of a shrinkage rule and application of the threshold to the detail coefficients. This can be accomplished by hard
- Application of the inverse transform (wavelet reconstruction) using the modified (threshold) coefficients.

## IV. THRESHOLDING

Thresholding is a technique used for signal and image de-noising. The shrinkage rule defines how we apply the threshold. There are two main approaches which are:

- **Hard thresholding**deletes all coefficients that are smaller than the threshold $\lambda$ and keeps the others unchanged. The hard thresholding is defined as follows:

$$\overline{c}_h(k) = \begin{cases} \text{sign } c(k) \ (|c(k)|) & \text{if } |c(k)| > \lambda \\ 0 & \text{if } |c(k)| \leq \lambda \end{cases} \quad (4.1)$$

Where λ is the threshold and the coefficients that are above the threshold are the only ones to be considered. The coefficients whose absolute values are lower than the threshold are set to zero.

– **Soft thresholding** (Fig. 4.1) deletes the coefficients under the threshold, but scales the ones that are left. The general soft shrinkage rule is defined by:

$$\overline{c}_s(k) = \begin{cases} \text{sign } c(k) \ (|c(k)| - \lambda) & \text{if } |c(k)| > \lambda \\ 0 & \text{if } |c(k)| \leq \lambda \end{cases} \quad (4.2)$$

For an illustration of what has been described above a linear signal is threshold according to the methods described using a threshold λ of 0.5 (Fig. 4.1).

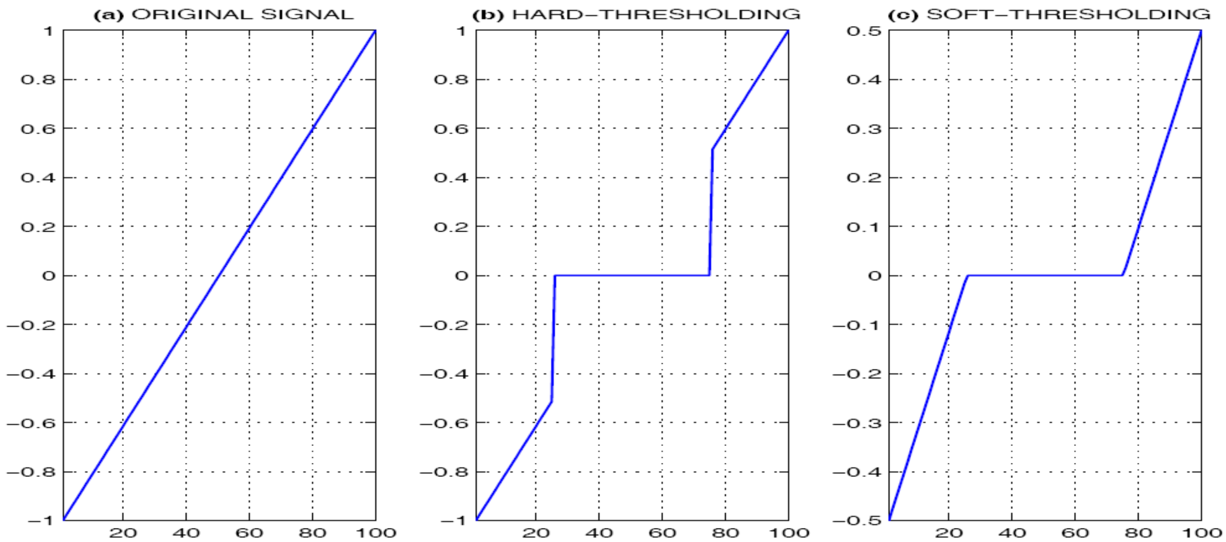


Fig. 4.1: An Example of (A) Linear Signal Threshold Using, (B) Hard-Thresholding, And (C) Soft-Thresholding

### *A. Global Threshold*

The global threshold method derived by Donoho is given by Eq. (4.3) has a universal threshold:

$$\lambda = \sigma\sqrt{2\log(N)} \quad (4.3)$$

Where N is the size of the coefficient arrays and $\sigma^2$ is the noise variance of the signal samples.

### *B. Level Dependent Threshold*

Level dependent thresholding method is done by using Eq. (5.4). Estimation of the noise standard deviation $\sigma_k$is done by using the robust median estimator in the highest sub-band of the wavelet transform

$$\lambda_k = \sigma_k\sqrt{2\log(N)} \quad (4.4)$$

Where the scaled MAD noise estimator is computed by:

$$\sigma_k = \frac{MAD_k}{0.6745} = \frac{(median(|\omega_i|))_k}{0.6745} \quad (4.5)$$

Where MAD is the median absolute deviation of the magnitudes of all the coefficients at the finest decomposition scale and $\omega_i$ are the coefficients for each given sub-band, the factor 0.6745 in the denominator rescales the numerator so that $\sigma_k$ is also a suitable estimator. The threshold estimation method is repeated for each sub-band separately, because the sub-bands exhibit significantly different characteristics.

### *C. Optimal Threshold Estimation*

Estimate the mean square error function to that compute the error of the output to minimize the function, the minimum MSE serves as a solution to the optimal threshold. A function of the threshold value which is minimized is defined in Eq. (4.6).

$$G(\lambda) = MSE(\lambda) = \frac{1}{N}||y - y_\lambda||^2 \quad (4.6)$$

If $y_\lambda$ is the output of the threshold algorithm with a threshold value λ and y is the vector of the clean signal, the remaining noise on this result equals $e_\lambda = y_\lambda - y$. As the notation indicates, the MSE is a function of the threshold value λ. Find the optimal value of λ that minimizes MSE (λ) and the convergence of the algorithm.

### *D. Measures of Image Quality*

One of the issues of de-noising is the measure of the reconstruction error. In order to separate the noise and image components from a single observation of a degraded image it is necessary to assume or have knowledge about the statistical properties of the noise. To get the measure of the wavelet filter performance, the experimental results are evaluated according to three error criteria namely, the mean square error (MSE) and the peak signal to noise ratio (PSNR). For most quality assessment methods, the error criterion takes the form of a Murkowski norm [31] which is defined as follows:

$$E(\{e_{m,n}\}) = \left(\sum_m \sum_n |e_{m,n}|^\beta\right)^{\frac{1}{\beta}} \quad (4.7)$$

Where $\{e_{m,n}\}$ is the error (difference) between the reference and de-noised image and β is a constant exponent typically chosen to lie between 1 and 4 for image error metrics. The goal of de-noising is starting from a noisy image to produce the best possible estimate ˜y (m, n) of the original image y (m, n). The measure of success in de-noising is usually an error measure E (ˆy(m, n), y (m, n)) between the original y(m, n) and the estimate ˆy(m, n). The mean square error (MSE) function is commonly used because it has a simple mathematical structure that is easy to compute and it is differentiable implying that a minimum can be sought. For a discrete image signal y (m, n) and its approximation (estimate) ˜y(m, n) where m, n = 0, 1, . . . , N − 1 the MSE is defined as

$$MSE = \frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left(y(m,n) - \tilde{y}(m,n)\right)^2 \quad (4.8)$$

Where y (m, n) and ˜y(m, n) represent the original image and the de-noised image respectively. The criterion root mean squared error (RMSE) is the square root of MSE, that means for RMSE β = 2.

Another most common and simplest measures of image quality, is the peak signal to noise ratio (PSNR) which is given by:

$$PSNR = 10\log_{10}\left(\frac{I_{max}^2}{MSE}\right) \quad (4.9)$$

Where $I_{max}$ is the maximum intensity value, typical PSNR values range between 20 and 40. They are usually reported to two decimal points (e.g. 25.47). An improvement in of the PSNR magnitude will increase the visual appearance of the image. PSNR is typically expressed in decibels (dB). For comparison with the noisy image the greater the ratio, the easier it is to identify and subsequently isolate and eliminate the source of noise.

## V. DISCRETE WAVELET TRANSFORM

By restricting to a discrete set of parameters we get the Discrete Wavelet Transform (DWT) [14] which corresponds to an orthogonal basis of functions all derived from a single function called the mother wavelet.

CWT is redundant since the parameters (a, b) are continuous thus it's necessary to discretize the grid on the time-scale plane corresponding to a discrete set of continuous basis functions. This lead us to a question: how can we discretize the wavelet in Eq. (5.1)

$$W_{j,k}(t) = \frac{1}{\sqrt{a_j}}\, W\left(\frac{t - b_k}{a_j}\right) \quad (5.1)$$

Wavelet analysis is simply the process of decomposing a signal into shifted and scaled versions of a mother (initial) wavelet. An important property of wavelet analysis is perfect reconstruction, which is the process of reassembling a decomposed signal or image into its original form without loss of information. For decomposition and reconstruction two types of basic functions normally used are:

- Scaling function Φjk(t)

$$\Phi_{jk}(t) = 2^{-\frac{j}{2}} \Phi_0(2^{-j} t - k) \quad (5.2)$$

- Wavelet $W_{jk}$ (t)

$$W_{jk}(t) = 2^{-\frac{j}{2}} \Psi_0(2^{-j} t - k) \quad (5.3)$$

Where m stands for dilation or compression and k is the translation index. Every basis function W is orthogonal to every basis function Φ. Wavelets are functions defined over a finite interval and have an average value of zero.

### *E. Filter Bank Decomposition*

The discrete wavelet transform (DWT) [15] is commonly implemented using dyadic multirate filter banks, which are sets of filters that divide a signal frequency band into sub bands. At each scale in DWT, the approximation coefficients are generated from a low pass filter and are associated with the low frequency trend while the detail coefficients are output from a high-pass filter and capture the high frequency components of the time series.

The inverse discrete wavelet transform (IDWT) reconstructs a signal from the approximation and detail coefficients derived from decomposition. The IDWT differs from the DWT in that it requires up sampling and filtering, in that order. Up sampling, also known as interpolating means the insertion of zeros between samples in a signal.

In Fig. 5.1, L and H represent the scaling function and wavelet function respectively. The wavelet filter coefficients are obtained from alternating flip of scaling filter coefficients. Short filter results in faster computation of convolutions. A pair of filters: a low-pass filter L and a high-pass filter H, split a signal's bandwidth in two halves. This provides the coefficients $c_j$ (k) and $d_j$ (k) for the decomposition of the signal into its scaling function and wavelet function components.

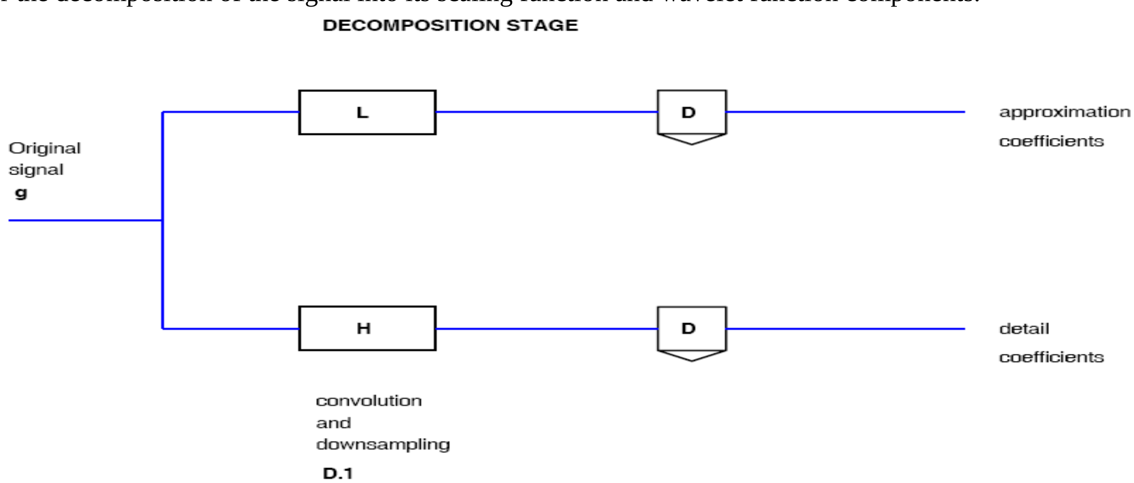


Fig. 5.1: One-Dimensional Signal Decomposition

### *F. Two-dimensional Discrete Wavelet Transform*

Digital images are 2-D signals that require a two-dimensional wavelet transform. The 2-D DWT analyzes an image across rows and columns in such a way as to separate horizontal, vertical and diagonal details. In the first stage the rows of an N × N are filtered using a high pass and low pass filters. This filtering is done using 1-D convolution with the coefficients $h_0$ (k) and $h_1$(k), since each row of the image is a one-dimensional signal. This is followed by down sampling with a factor of 2 which removes every odd numbered sample in the filtered result this has the effect of removing every other column of the N × N block giving an N × (N/2) image.

In the second stage 1-D convolution with $h_0$ (k) and $h_1$(k)is applied to the columns of the filtered image. Down sampling removes each odd-numbered sample in each column of the now twice-filtered result which results in the removal of every other row. Each of the branches in the tree is shown in the Fig. 5.2 therefore produces an (N/2) × (N/2) sub image. This leads at each level to 4 different sub bands HH, HL, LH and LL. The LL is filtered again to get the next level representation; Fig. 5.2 summarizes the transform for a one level decomposition.

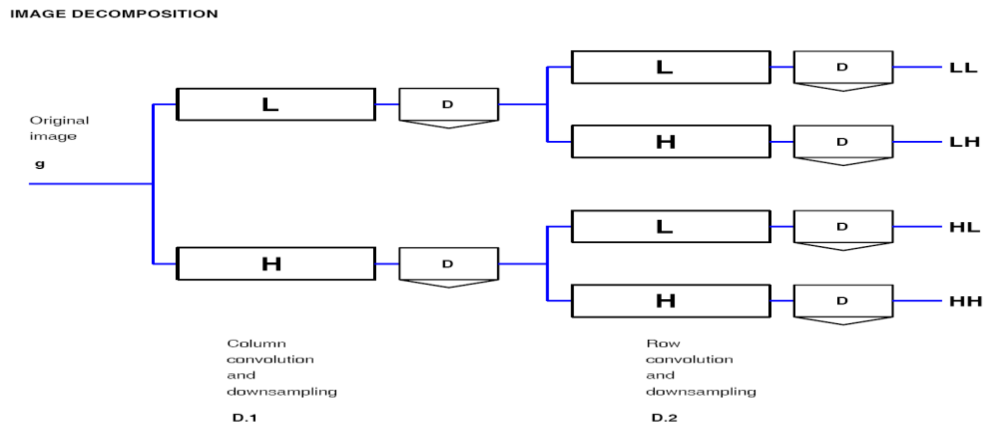


Fig. 5.2(A) : One-Level Two-Dimensional DWT Decomposition

### *G. Reconstruction*

To reconstruct the image from its 2-D DWT sub images (LH, HL, HH) the details are recombined with the low pass approximation using up sampling and convolution as shown in Fig. 5.3. Up sampling refers to the insertion of a zero row after each existing row or a zero column after each existing column. In the first stage the columns of the up sampled sub images are convolved with the impulse responses $h^T_0$ (k) and $h^T_1$ (k) and in the second stage the rows of the up sampled sums are convolved with the same impulse responses.

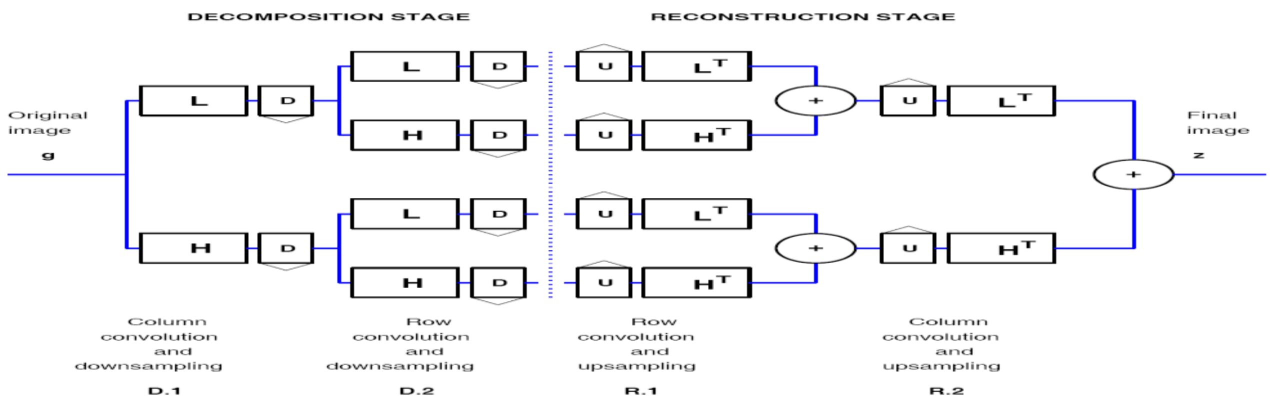


Fig. 5.3: Two-Dimensional DWT Reconstruction

## VI. RESULTS

### *A. Result from Real CT Data - 2-D*

We have done simulations with uniform random noise added to the CT image. An example of a noisy computed tomography image (CT) which consists of 128 × 128 pixels is shown in Fig. 6.5. As can be seen in the background the image has been uniformly corrupted with additive noise. The de-noising techniques discussed in the previous section are applied to the noisy CT image to test the efficiency of the different threshold methods.

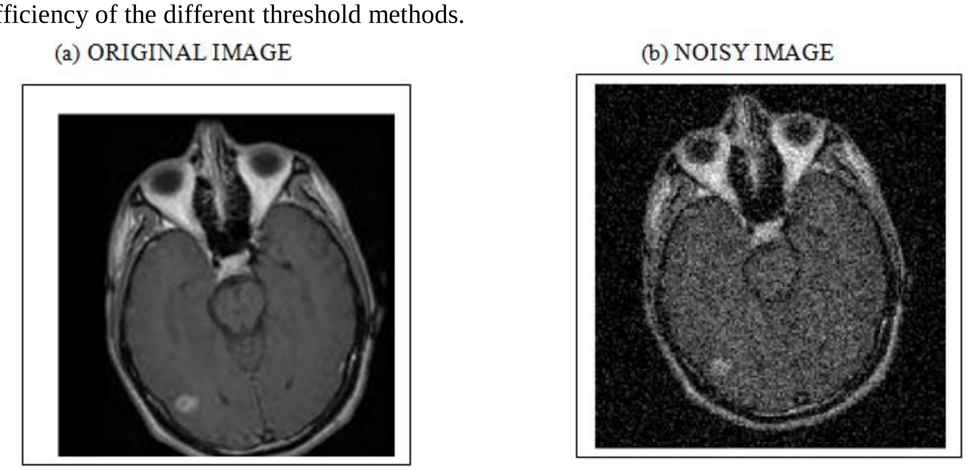


Fig. 5.5: (A) Original Image And (B) Noisy Image

For comparison of the five different wavelet functions, the quantitative de-noising results of the CT images obtained by using global, level-dependent and optimal thresholding are shown in Tabs. 5.4, 5.5 and 5.6 respectively. The MSE, MAE, PSNR error criteria are the ones which have been used to assess the performance of the wavelet functions. Their numerical results are summarized in the tables.

– It is clear from the table 5.3, for Global thresholding technique; sym4 gives best result for level-1 & db4 performs well for level-2.

Table - 5.3
Qualitative Analysis (Ct Image) - Global Thresholding

| *Type of wavelet* | *LEVEL 1* | | *LEVEL 2* | |
|---|---|---|---|---|
| | *MSE* | *PSNR(db)* | *MSE* | *PSNR(db)* |
| *Haar* | *0.09204826* | *26.32579* | *0.9504819* | *24.32571* |
| *bior1.1* | *0.09346032* | *25.34704* | *0.9646020* | *24.34712* |
| *bior 1.3* | *0.09856421* | *28.78082* | *0.9856421* | *28.69082* |

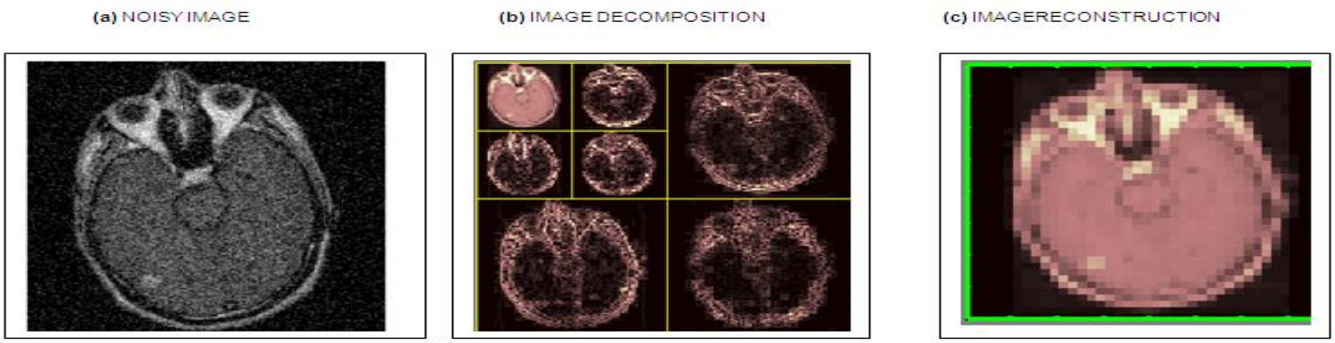


Fig. 5.6 : The 2-D Image Decomposition of The (A) Noisy CT Image Using A Db4 Wavelet Function, (b) the approximation image (low-frequency component) is in the top-left corner of the
Transform display, the other sub images contain the high frequency details (c) the resulting de-noised image, obtained by taking the inverse thresholded coefficients. (d) Global thresholding of the sub band coefficients

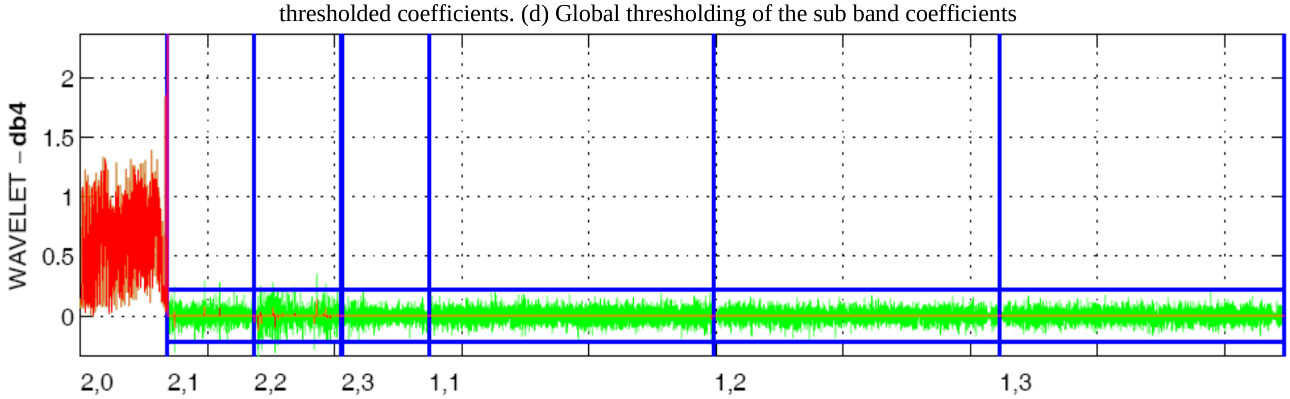


Fig. 5.7: Global Thresholding of The Sub Band Coefficients

– It is clear from the table 5.5, for Level Dependent thresholding technique; sym2 gives best result for level-1 & db2 performs well for level-2.

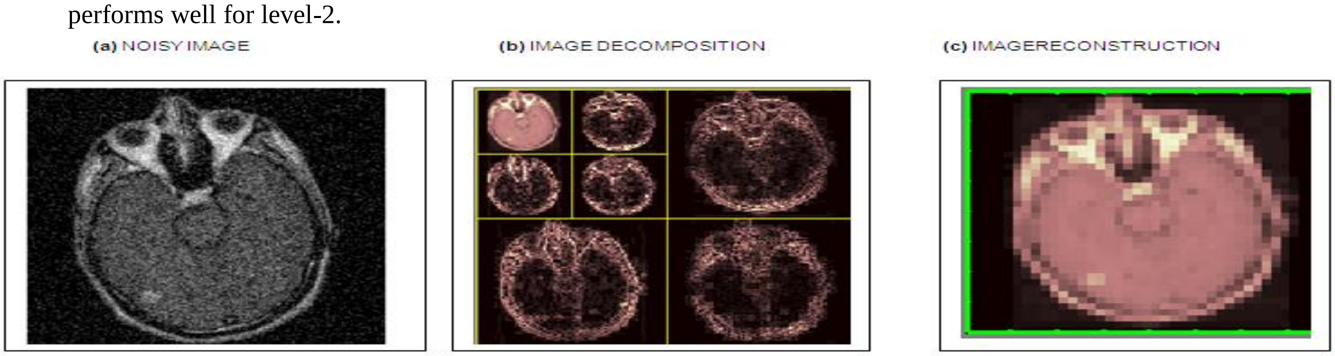


Fig. 5.8: Discrete Wavelet Transform Of The (A) Noisy CT Image Using A Db2 Wavelet Function, (B) Two-Level Image Decomposition, (D) Level Dependent Thresholding Of The Sub Band
Coefficients, And (C) The Resulting De-Noised Image, Obtained By Taking The Inverse Threshold Coefficients.

### *B. Scaling and Wavelet Coefficient Optimal thresholding Brain*

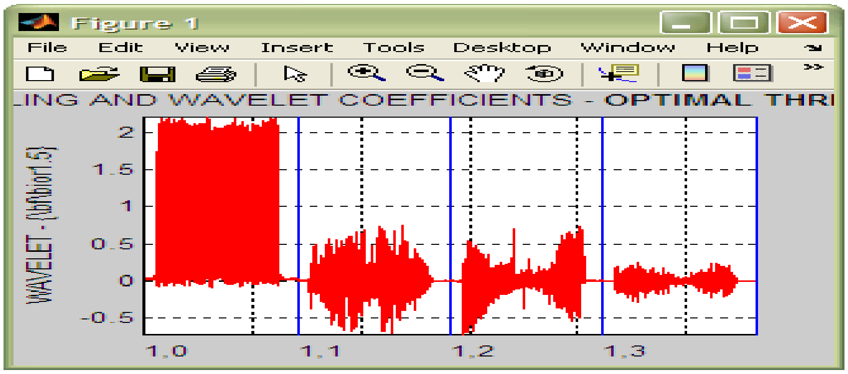


Fig. 5.9: Scaling And Wavelet Coefficient Optimal Thresholding of Brain

### *C. Scaling and Wavelet Coefficient Optimal thresholding of Lungs*

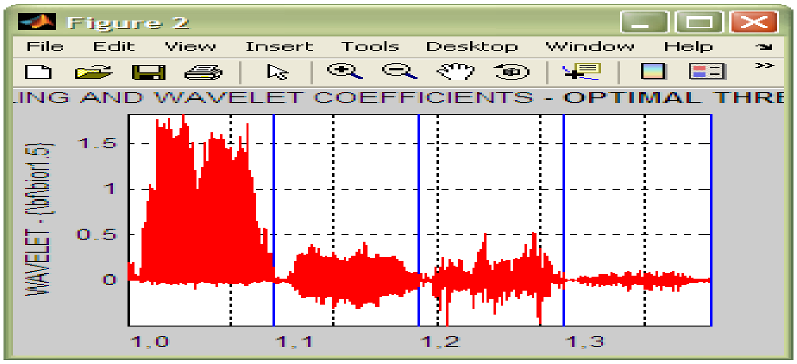


Fig. 5.10: Scaling And Wavelet Coefficient Optimal Thresholding of Lungs

– It is clear from the table 5.5, for Optimal thresholding technique; bior1.3 gives best result for both level-1 & level -2.

Table - 5.6
Qualitative Analysis (Ct Image) - Optimal Thresholding

| *Type of wavelet* | *LEVEL 1* | | *LEVEL 2* | |
|---|---|---|---|---|
| | *MSE* | *PSNR(db)* | *MSE* | *PSNR(db)* |
| *Haar* | *0.9804826* | *28.32579* | *0.9804819* | *28.32571* |
| *bior1.1* | *0.9846032* | *28.34704* | *0.9846020* | *28.34712* |
| *bior 1.3* | *0.9856421* | *28.78082* | *0.9856421* | *28.78082* |

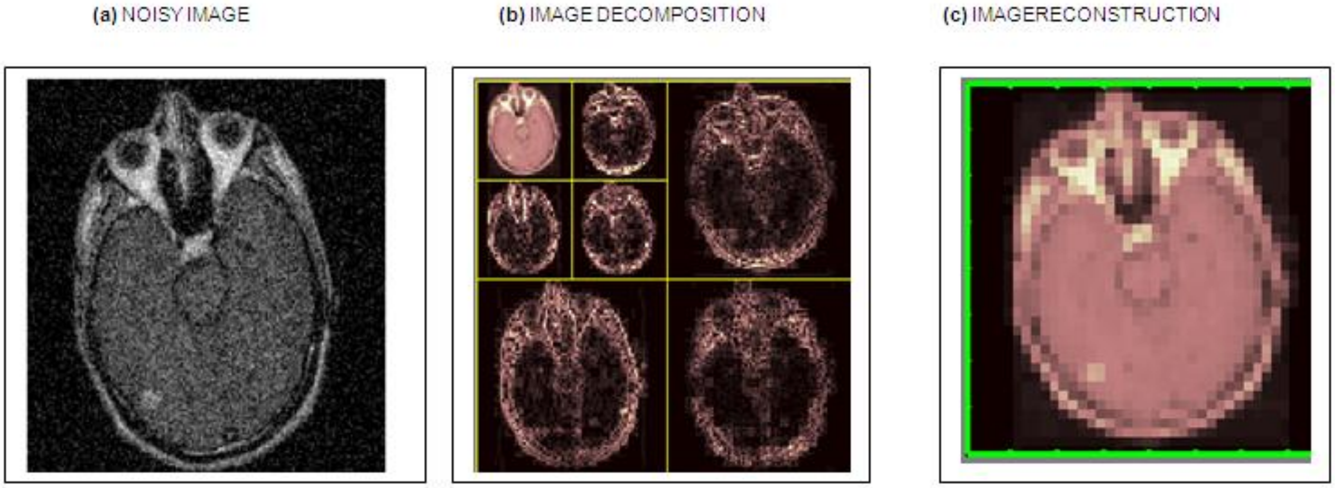


Fig. 5.11: Discrete Wavelet Transform Of The (A) Noisy CT Image Using A Bior1.3 Wavelet Function, (B) The Decomposed Image Showing The Approximation Image And The Detail Sub Band Sub Images And (C) Shows The De-Noised CT Image.

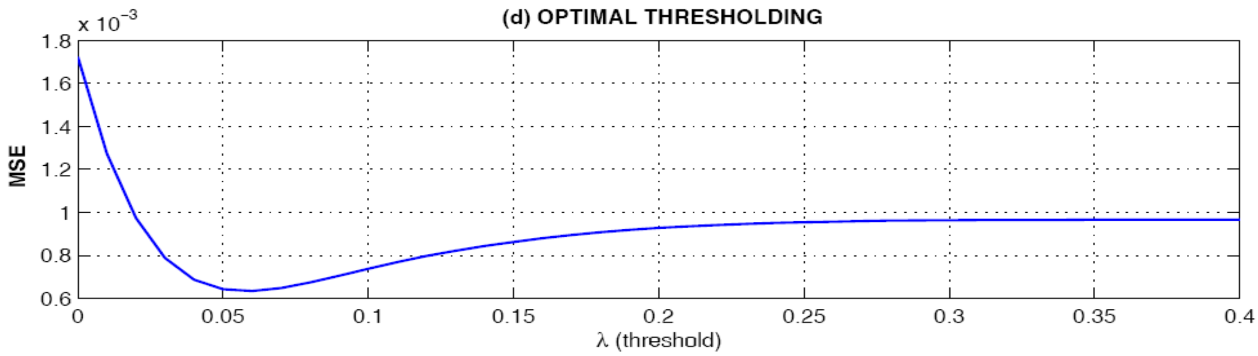


Fig. 5.12: Optimal Thresholding Showing The Optimum Threshold Which Minimizes The MSE,

- From the comparison results it can be observed, that the bior1.3 wavelet & optimal thresholding technique gives greatly improved de-noising results for both level-1 & level-2.

Hence from the above tables, we observed that for both Simulated& CT Image, bior wavelet & Optimal Thresholding technique gives the best denoised results. Its gives higher PSNR & lower MSE value.

## VII. CONCLUSIONS

The de-noising process consists of decomposing the image, thresholding the detail coefficients, and reconstructing the image. The decomposition procedure of the de-noising example is accomplished by using the DWT. Wavelet thresholding is an effective way of de-noising as shown by the experimental results obtained with the use of different types of wavelets. Thresholding methods implemented comprised of the universal global thresholding, level (sub- band) thresholding and optimal thresholding. More levels of decomposition can be performed; the more the levels chosen to decompose an image, the more detail coefficients we get. But for de-noising the noisy MR data sets, two-level decomposition provided sufficient noise reduction.

In this thesis we have presented the generalization of the DWT method for the 2-D case. The resulting algorithms have been used for the processing of noisy MR image. Experimental results have shown that despite the simplicity of the proposed de-noised algorithm it yields significantly better results both in terms of visual quality and mean square error values. Considering the simplicity of the proposed method, we believe these results are very encouraging for other forms of de-noising. The Biorthogonal wavelet (bior1.1) & Biorthogonal wavelet (bior1.3) gave the best results compared to other wavelets for both Simulated & MRI image respectively. Optimal thresholding gives better denoised result among the three thresholding technique. Finally, a great advantage of the wavelet transform is that often a large number of the detail coefficients turns out to be very small in magnitude, truncating (removing) these small coefficients from the representation introduces only small errors in the reconstructed image, giving an image which closely resembles the original image and also preserving edge features.

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