Partial Transmit Sequence Scheme For PAPR Reduction In OFDM Systems: A Review

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Abstract

The high Peak to Average Power Ratio (PAPR) of the transmitted signals is the main drawback of OFDM system. OFDM Suffers as the no of Subcarriers operating in the large dynamic range operates in the non-linear region of amplifier due to OFDM suffer the PAPR problem. Application of high power amplifiers results in increased component cost. To reduce Peak-to-average power ratio (PAPR) in orthogonal frequency division multiplexing (OFDM) Partial transmit sequence scheme is a promising algorithm. Partial Transmit Sequence (PTS) consist of several inverse fast Fourier transform (IFFT) operations and complicated calculations to obtain optimum phase sequence which results in increasing the computational complexity of PTS. There are various methods and scheme has been proposed to reduce the complexity of the algorithm with no increment or reduction in the PAPR also. This article presents a review of various complexity reduction techniques for PTS.

Keywords: PAPR – Peak-to-Power ratio, OFDM, PTS.

I. INTRODUCTION

In broadband wireless communications high bit rate transmission is required for high quality communications. OFDM is a very attractive technique for high speed data transmission over multipath fading channels. OFDM technique has been applied extensively to digital transmission, such as in wireless local area networks and digital video/audio broadcasting systems. Moreover, it has been regarded as a promising transmission technique for fourth-generation wireless mobile communications.

However, due to its multicarrier nature, the major drawback of the OFDM system is the high peak-to-average power ratio (PAPR), which may cause high out-of-band radiation when the OFDM signal is passed through a radio frequency power amplifier. Consequently, the high PAPR is one of the most important implementation challenges that face designers of OFDM [1].

The PAPR problem is one of the most important issues for developing multicarrier transmission systems. Various methods for PAPR reduction have been proposed in the literature [2]–[6], to avoid the occurrence of high PAPR of OFDM signals. Among these methods, the partial transmit sequence (PTS) technique [6] is the most attractive scheme because of good PAPR reduction performance and no restrictions to the number of the subcarriers. In the PTS scheme, the input data is divided into smaller disjoint subblocks. Each subblock is multiplied by rotating phase factors. The subblocks are then added to form the OFDM symbol for transmission. The objective of the PTS scheme is to design an optimal phase factor for a subblock set that minimizes the PAPR.

The PTS technique significantly reduces the PAPR, but unfortunately, finding the optimal phase factors is a highly complex problem. To reduce the complexity of PTS several methods have been proposed. In this paper we have discussed those methods in regards to basic PTS technique.

II. OFDM SYSTEM FOR WIRELESS COMMUNICATION

In an OFDM system, the total signal frequency band is divided into N non-overlapping frequency sub channels. Each sub channel is modulated with a separate symbol, and then the N sub channels are frequency multiplexed. It seems good to avoid spectral overlap of channels to eliminate inter-channel interference. However, this leads to inefficient use of the available spectrum. To cope with the inefficiency, the ideas proposed in the mid-1960s were to use parallel data and FDM with overlapping sub channels, in which each, carrying a signalling rate b, is spaced b apart in frequency to avoid the use of high-speed equalization and to combat impulsive noise and multipath distortion, as well as to use the available bandwidth fully. By using the overlapping multicarrier modulation technique, almost 50% of bandwidth can be saved. Basic Principal of OFDM system [3] is to divide high data rate transmission into lower data rate and that are transmitted simultaneously over number of
subcarriers. Each of these signal are individually modulated and transmitted over the channel. And at the Receiver and signal will be demodulated and recombine to recover the Original Signal. As shown in figure given below each subcarrier arranged orthogonally in spectrum. Periodic signal are orthogonal when integral of their product is zero. Fig. 1 shows the block diagram of the OFDM system.

III. OFDM SYSTEM MODEL AND PROBLEM DEFINITION

A. OFDM System Model and PAPR Definition

In an OFDM system with \( N \) subcarriers, the discrete-time transmitted OFDM signal is given by:

\[
x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi nk/N}, \quad k = 1, 2, \ldots, PN - 1
\]  

(1)

where \( j = \sqrt{-1}, X_n, n = 0, 1, \ldots, N - 1 \) are input symbols modulated by PSK or QAM, and \( P \) is an integer that larger or equal 1 called over-sampling factor. When \( P = 1 \), the samples are achieved by use of the Nyquist rate sampling. We shall write the input data block as a vector, \( \mathbf{X} = [X_0, X_1, \ldots, X_{N-1}] \). The PAPR of the transmitted signal in (1), defined as the ratio of the maximum to the average power, can be expressed by

\[
\text{PAPR} = 10\log_{10} \frac{\max |x_k|^2}{E[|x_k|^2]} \text{ (dB)}
\]

(2)

where \( E[\cdot] \) denotes the expected value operation.

PAPR is a random variable because it is a function of the input data, and the input data are random variable. Therefore PAPR can be calculated by using level crossing rate theorem that calculates the average number of times that the envelope of a signal crosses a given level. Knowing the amplitude distribution of the OFDM output signals, it is easy to compute the probability that the instantaneous amplitude will be above a given threshold and the same goes for power. This is performed by calculating the complementary cumulative distribution function (CCDF) for different PAPR values as follows:

\[
\text{CCDF} = \Pr(\text{PAPR} > \text{PAPR}_0)
\]

(3)

B. OFDM System With PTS to Reduce the PAPR

As shown in Fig. 1, in a typical OFDM system with a PTS scheme to reduce PAPR, the input data \( \mathbf{X} \) is partitioned into smaller disjoint subblocks \( \mathbf{M} \), which are represented by the vector \( \mathbf{X}_m \), where \( m = 1, 2, \ldots, M \), such that:

\[
\mathbf{X} = \sum_{m=1}^{M} \mathbf{X}_m.
\]

(4)

Here, it is assumed that each subblock consists of a set of sub carriers of equal size. Next, the partitioned subblocks are converted from the frequency domain to the time domain using the N-point inverse fast Fourier transform (IFFT). Since the IFFT is a linear transformation, the representation of the block in time domain is given by:
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Let \( x = \text{IFFT} \left\{ \sum_{m=1}^{M} x_m \right\} \) be the time domain signal after combination given by

\[
x' (b) = \sum_{m=1}^{M} b_m \cdot x_m.
\]

The allowable phase factors are

\[
b_m = e^{j\phi_m}
\]

Where \( \phi_m \) can be chosen freely within \([0, 2\pi]\). For convenience, (5) can be expressed as

\[
x' (\Phi) = \sum_{m=1}^{M} e^{j\phi_m} \cdot x_m,
\]

where \( \Phi = [\phi_1, \phi_2, ..., \phi_M]^T \). Hence, the objective of the PTS scheme is to design an optimal phase factor for the subblock set that minimizes the PAPR. Thus, the minimum PAPR with the PTS technique is related to the problem

\[
\text{minimize } \max |\mathbf{x}' (\Phi)|
\]

subject to \( 0 \leq \phi_m < 2\pi, \quad m = 1, 2, \ldots, M. \)

It is obvious that finding a best phase factor set is a complex and difficult problem; therefore, in the next section, we propose a novel implementation of the PTS scheme based on the CE method.

IV. VARIOUS METHODS FOR COMPLEXITY REDUCTION OF PTS TECHNIQUE

There are various approaches have been followed for the reduction in the PAPR along with the complexity of the PTS technique. For this purpose, either some changes has been made in the algorithm or some phase sequence has been altered.

A. Block Partition Based PTS Techniques

1) Interleaved sub-block partition based PTS

Let \( x = [X(0) X(1) \ldots \ldots X(N_0 - 1)] \) be the OFDM input symbol sequence with length \( N_0 \). And \( X_0 \ldots \ldots X_{M-1} \) are the OFDM symbols after portioning and the \( X_0 \ldots \ldots X_{M-1} \) with length \( N_0 \) after zero padding. Consider that the interleaving partition method is used in PTS OFDM scheme, \( X_m' \) would be expressed as

\[
X_m' = [X(mN), X(M + m), \ldots \ldots X((N-1)M + m)]_{1:N_0}
\]

and \( X_m \) would be expressed as

\[
X_m' = [0, 0, \ldots \ldots 0, X(mN), 0, \ldots \ldots 0, X(M + m), 0, \ldots \ldots 0, X((N-1)M + m)]_{1:N_0}
\]

where \( 0 \leq m \leq M - 1 \) and \( N = N_0 / M \). It is clear that the most elements of \( X_m' \) are zeros, therefore there are many unnecessary multiplications and additions to zeros while applying \( N_0 \times N_0 \) IFFT would be replaced by \( N \times N_0 \) IFFT [9].

2) Adjacent sub-block partition based PTS

In this scheme, the complexity of PTS scheme would be reduced by eliminating these multiplications and additions in figure 1. Consider that the adjacent partition method is used in PTS OFDM scheme,

\[
X_m' = [X(mN)X(mN + 1), \ldots \ldots X(N(M + m - 1))]_{1:N_0}
\]

and

\[
X_m' = [0, 0, \ldots \ldots 0, X(mN), \ldots \ldots 0, X(NM + m), \ldots \ldots 0, X((N-1)M + m), \ldots \ldots 0]_{1:N_0}
\]

where \( 0 \leq m \leq M - 1 \) and \( N = N_0 / M \).

In fig 1, the complexity reduction of partial transmit sequence (PTS) PAPR reduction scheme in OFDM systems by reducing the complexity of the IFFT architecture are investigated in this scheme. In the IFFT architecture of PTS OFDM scheme, there are
a lot of additions and multiplications with zero, which are obviously unnecessary. We can efficiently reduce the computational complexity without changing the resulting signal or degrading the performance of PAPR reduction by eliminating the addition and multiplications with zero from the architecture [9].

3) Sub-optimal sub-block partition based PTS
In the fig 1, there are steps to be taken in sub-optimal method, these are as follows:

1) Set, \( b_m = 1 \), \( m = 1, 2 \ldots M \) using (2) and (3), we can calculate PAPR of OFDM signals with the value of PAPR1, and set \( index = 1 \);
2) Set \( b\text{index} = 1 \), PAPR at this time is calculated by the same method with the value of PAPR2.
3) If PAPR1 > PAPR2, \( b\text{index} = 1 \); otherwise PAPR1 = PAPR2, 1 index \( b\text{index} = 1 \);
4) \( index = index + 1 \);
5) Repeat from step 2-4 if \( index < M + 1 \).

The amount of computation can be effectively reduced by sub-optimal PTS algorithm. Compared to \( 2^M-1 \) IFFT operations of optimum PTS, the computational cost of PTS is only \( M \) IFFT operations [10].

B. The CE Algorithm Based PTS Technique
In the PTS approach, the objective is to find the phase factors with the aim of minimizing the PAPR. However, in order to employ the CE method to find the phase factors that minimize the PAPR in the PTS technique, we have to define the score function for the proposed CE algorithm. We define the inverse of the PAPR as the evaluation function such that its value increases as the PAPR decreases. Hence, in the proposed CE algorithm-based PTS approach, we are interested in maximizing the score function. The CE method is an adaptive importance sampling method that transforms the deterministic optimization problem into a family of stochastic sampling problems. Hence, the first step in using the CE method is to randomize our original deterministic problem by including a set of sampling distribution over deterministic.

C. Altered Phase Sequence Based PTS Techniques
There are various different phase alteration techniques and Different Phase sequences have been proposed for the reduction in complexity and PAPR.

In order to decrease the complexity of conventional PTS, in [11] a new phase sequence has been generated. This new phase sequence is based on the generation of \( N \) random values of \( \{1 -1\} \). If we consider the number of allowed phase factors is \( W = 2 \). Hence the new phase subsequence has a formation as follows:

\[
\hat{\mathbf{B}} = \begin{bmatrix}
\hat{b}_{1,1} & \hat{b}_{1,2} & \ldots & \hat{b}_{1,N} \\
\vdots & \vdots & \ddots & \vdots \\
\hat{b}_{M/2,1} & \hat{b}_{M/2,2} & \ldots & \hat{b}_{M/2,N} \\
\end{bmatrix}_{M/2 \times N}
\]

(9)

According to (9), \( N \) random phase sequences are generated and periodically \( M/2 \) times will be generated where \( M \) is the number of subblock partitioning.

In [11] author has use random phase sequence matrix is given in (10). For computing the actual PAPR, the oversampling needs to be considered. Hence the matrix can be expressed as:

\[
\hat{\mathbf{B}} = \begin{bmatrix}
\hat{b}_{1,1} & \ldots & \hat{b}_{1,NL} \\
\vdots & \ddots & \vdots \\
\hat{b}_{M/2,1} & \ldots & \hat{b}_{2,NL} \\
\hat{b}_{M/2+1,1} & \ldots & \hat{b}_{M/2+1,NL} \\
\vdots & \ddots & \vdots \\
\hat{b}_{D,1} & \ldots & \hat{b}_{D,NL} \\
\end{bmatrix}_{(D+NL) \times 1}
\]

(10)

where \( L \) is the oversampling factor. It should be noted that in order to have exact PAPR calculation, at least 4 times oversampling is necessary.

Further explanation is given in [12]. The new phase sequence is based on the generation of \( N \) random values from the possible phase factors \( \{1 -1\} \), if we consider the number of allowed phase factor \( W=2 \). Therefore the new phase sequence can be constructed as follows:
In conventional PTS, it was mentioned that for searching the optimum phase factor, \(W^{V-1}\) iteration is required. It means that, if for example \(W=2\) and \(V=4\), then we need 8 iterations to find the optimum phase sequence. For the new phase sequence format, the way to find the optimum phase factor will be different. In this case, first \(N\) different random phase sequence is generated and this is continued \(V\) times according to (9), hence the optimum phase factor is each row of this matrix. But for finding the optimum phase factor, matrix in (9) should be randomly generated several times. We constrain the number of times that the matrix would be generated to be the same as in C-PTS for fair comparison. Hence for the case of \(W=2\) and \(V=4\), the C-PTS has 8 iterations and therefore (9) should be generated 8 times. For example in this case we have 8 possibilities, because the first bit is fixed, \{1,1,1\}, \{1,1,-1\}, \{1,-1,1\}, \{1,-1,-1\}, \{-1,1,1\}, \{-1,1,-1\}, \{-1,-1,1\}, \{-1,-1,-1\}. The optimum phase factor will be chosen from these 8 phase sequences. In our proposed method, because there are \(N\) different random phase factors, to search for the optimum phase sequence it requires \(N\times8\) iterations which is not practical. But here, we only apply the same iteration as was applied in C-PTS and later it will be shown through simulations, that good PAPR performance is achieved, and it is also possible to have less iteration while keeping the PAPR performance the same as C-PTS but with reduced complexity.

\[
\hat{b} = \begin{bmatrix}
    b_{1,1}, & b_{1,2}, & \cdots, & b_{1,N} \\
    \vdots & \vdots & \ddots & \vdots \\
    b_{V,1}, & b_{V,2}, & \cdots, & b_{V,N}
\end{bmatrix}_{P\times N} 
\]  

(11)

where \(P\) is the number of iterations that should be set in accordance with the number of iterations of the C-PTS. Where \(P>V\) and is the number of iterations that should be set in accordance with the number of iterations of the PTS. The value of \(P\) can be calculated as follows:

\[P = D W^{V-1}\]  

(13)

And \(D = 1, 2, \ldots, D_N\)

where \(D\) is the coefficient that can be specified based on the PAPR reduction and complexity and \(D_N\) is the amount that is specified by user. The value of \(P\) explicitly depends on the number of subblocks \(V\) if assuming the number of allowed phase factor is constant.

D. Low Complexity PTS with Clipping

In this method, the OFDM symbols are divided into number of sub blocks and the signals corresponding to IFFT of each sub block is clipped. This method provide better PAPR reduction than the PTS scheme with new phase sequence but the clipping introduces slight distortion in the signal.

![Block Diagram of Low Complexity PTS With Clipping](image)

Fig. 4: Block Diagram of Low Complexity PTS With Clipping

Here, the OFDM sequence is divided into several \(V\) sub blocks as in PTS and new phase sequence applied to PTS schemes. In this method separate clipping is done on each block. The peak clipping can be performed only on time domain signal. Hence
clipping is introduced in the new phase sequence applied to PTS scheme after the IFFT operation in each block. The advantages of this scheme are:

- Low complexity: As clipping is a PAPR reduction technique which is simple in implementation, the introduction of clipping to a low complexity PTS scheme does not increase the complexity of the system.
- Better PAPR reduction: The combined effect of clipping and PTS with new phase sequence scheme reduces PAPR considerably.
- Enhancement of power efficiency and therefore less power consumption and more battery life.

V. CONCLUSION

In this paper we have seen the Basics of OFDM. We have seen the OFDM system model and problem in the OFDM system. We have also seen the definition and significance of PAPR in OFDM. Then we have discussed the PTS technique in detail. Further we have discussed the various technique for the Complexity reduction of PTS technique. We have studied the Block partition based methods, the CE Algorithm Based PTS Technique, Altered Phase Sequence Based PTS Techniques and Low Complexity PTS with Clipping.

In [13] a method is used in which the Altered Phase Sequence Based PTS Techniques and Low Complexity PTS with Clipping is used in a combined way. We may implement the same technique for the SLM and this would be the base of our future work.

REFERENCES