Approximate Solution of the MHD Boundary Layer Flow Pasta Moving Continuous Flat Surface using Quartic Spline

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Abstract

To solve the equation of Magneto hydrodynamic boundary layer flows past a moving continuous flat surface in the presence of transverse magnetic field. The governing nonlinear equations and their associated boundary conditions are transformed into linear differential equation and solve it using quartic spline collocation method. It is shown that a class of similarity equations is available if free stream velocity varies along X- coordinate. Numerical and graphical presentation of flow problem is also given.
Keywords: Quartic Spline collocation, Third order ordinary differential equation, Quasilinearization, Upper triangular matrix, Linear equations

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I. INTRODUCTION

Magneto hydrodynamics is the study of the motion of an electrically conducting fluid in the presence of a magnetic field. Due to the motion of an electrically conducting fluid in a magnetic field the electrical currents are induced in the fluid which produces their own magnetic field, called induced magnetic field, and these modify the original magnetic field. In addition to this the induced currents interacts with the magnetic field to produce electromagnetic forces perturbing the original motion. Thus the two important basic effects of Magneto hydrodynamics are (i) the motion of the fluid affects the magnetic field and (ii) the magnetic field affects the motion of the fluid. Boundary layer flow of an electrically conducting fluid over moving surfaces emerges in a large variety of industrial and technological applications. It has been investigated by many researchers; [1]Wu (1973) has studied the effects of suction or injection in a steady two-dimensional MHD boundary layer flow of on a flat plate.[2] Takhar et.al. (1987) studied a MHD asymmetric flow over a semi-infinite moving surface and numerically obtained the solutions.[3] Mahapatra and Gupta (2001) treated the steady two- dimensional stagnation-point flow of an incompressible viscous electrically conducting fluid towards a stretching surface, the flow being permeated by a uniform transverse magnetic field.[4] Jean-David Hoernel (2008) has been investigated the similarity solutions for the steady laminar incompressible boundary layer governing MHD flow near forward stagnation-point of two-dimensional and axisymmetric bodies. [5] The MHD boundary layer flow past a flat plate was discussed in detailed, probably first time by Rossow (1957). Heat transfer of this case has been discussed by Rossow (1957), [6] Carrier and Green Span (1959), and [7]Afzal (1972).Rossow (1957) has considered transverse magnetic field whereas Carrier et al. (1959) have studied the effect of a longitudinal magnetic field effect on the velocity and the temperature distribution but all these cases are limited to the geometry of two dimensional flows.[8] Manisha et al (2009) had discussed steady two dimensional stagnation point flow of an incompressible viscous electrically conducting fluid over a flat plate.

The governing equations here are highly nonlinear differential equations, which are solved by using the Quartic spline collocation method. In this way, the paper has been organized as follows. In section 2, we use the Quartic spline collocation method. Section 3, approximate solution for the governing equations and contains the results and discussion. The conclusions are summarized in section 4.

II. QUARTIC SPLINE COLLOCATION METHOD

Consider equally spaced knots of partition π: $a = x_0 < x_1 < x_2 < .... < x_n = b$ on $[a,b]$. The quartic spline is defined by

$$s(x) = a_0 + b_0(x-x_0) + \frac{1}{2}c_0(x-x_0)^2 + \frac{1}{6}d_0(x-x_0)^3 + \frac{1}{24}\sum_{k=0}^{n-1} e_k (x-x_k)_+^4 \qquad (1)$$

Where the powers function $(x-x_k)_+$ is defined as

$$(x-x_k)_+ = \begin{cases} x-x_k, & x > x_k \\ 0, & x \le x_k \end{cases} \qquad (2)$$

and the boundary value problem is given by

$$y'''(x)+p(x)\,y''(x)+q(x)\,y'(x)+r(x)\,y(x)=m(x) \tag{3}$$

Subject to boundary conditions

$$\alpha_0\, y_0 + \beta_0\, y_n' + \gamma_0\, y_n'' = \delta_0$$
$$\alpha_1\, y_0' + \beta_1\, y_n + \gamma_1\, y_n'' = \delta_1$$
$$\alpha_2\, y_0'' + \beta_2\, y_n + \gamma_2\, y_n' = \delta_2$$

To solve this boundary value problem substitute $s(x)$, $s'(x)$, $s''(x)$, $s'''(x)$ from quartic spline, then the boundary value problem becomes

$$\sum_{k=0}^{n-1} e_k \left\{ (x_i - x_k)_+ + \frac{1}{2} p_i (x_i - x_k)_+^2 + \frac{1}{6} q_i (x_i - x_k)_+^3 + \frac{1}{24} r_i (x_i - x_k)_+^4 \right\}$$

$$+ d_0 \left\{ 1 + p_i (x_i - x_0) + \frac{1}{2} q_i (x_i - x_0)^2 + \frac{1}{6} r_i (x_i - x_0)^3 \right\}$$

$$+ c_0 \left\{ p_i + q_i (x_i - x_0) + \frac{1}{2} r_i (x_i - x_0)^2 \right\}$$

$$+ b_0 \left\{ p_i + r_i (x_i - x_0) \right\} + a_0 \{ r_i \} = m\{x_i\}$$ .Where i = 0,1,2,.....n.

Thus for quartic spline and third order boundary value problem we get nine linear algebraic equations in nine unknowns a_0, b_0, c_0, d_0, e_0, e_1,.... e_4. The matrix form of this system is given by

$$AX = B$$

Where $X = [e_4, e_3, e_2, e_1, e_0, d_0, c_0, b_0, a_0]^T$

$$B = [\delta_2, \delta_1, \delta_0, m_5, m_4, m_3, m_2, m_1, m_0]^T$$

And the co-efficient matrix A is an upper Hessenberg matrix.

Quartic spline and third order boundary value problem, we take number of intervals n=5. Continuing this process we can say that "In general for higher degree spline and lower order boundary value problem i.e. for n^{th} degree spline and $(n-1)^{th}$ order boundary value problem we can take (n+1) number of intervals at x_i, $i = 0(1)\,n$ and get a set of linear algebraic equations in (2n-1) unknowns.

III. Solution by using Collocation method

[9] Srivastava et al (1987), the basic equations governing the motion of two dimensional, steady incompressible viscous fluids past continuous surface in the presence of transverse magnetic field can be written in non-linear coupled equation as follow:

$$f''' - \beta f' + f f'' = 0, \tag{4}$$

If, $\beta = 0$ i.e. for non-magnetic case. Subject to boundary conditions as $f'(0) = 1,\ f(0) = 1,\ f'(0.5) = 0.$ (5)

We use quasilinearization technique to convert (4) into linear form with help of boundary conditions (5).

We get linear form as

$$f_i''' + f_i\, f_{i+1}'' - \beta f_{i+1}' + f_i''\, f_{i+1} = f_i''\, f_i \tag{6}$$

With boundary conditions as (5). The Quartic spline is given by

$$s(\eta) = a_0 + b_0(\eta - \eta_0) + \frac{1}{2} c_0 (\eta - \eta_0)^2 + \frac{1}{6} d_0 (\eta - \eta_0)^3 + \frac{1}{24} \sum_{k=0}^{n-1} e_k (\eta - \eta_k)_+^4 \tag{7}$$

After applying quasilinearization technique, we get linear differential equation (6).

Solve above equation and substitute constants in (7) and we get the solution for different values of β .

$$\sum_{k=0}^{n-1} e_k[(\eta_i-\eta_k)+\frac{f_i}{2}(\eta_i-\eta_k)^2-\frac{\beta}{6}(\eta_i-\eta_k)^3+\frac{f_i^{'}}{24}(\eta_i-\eta_k)^4]$$
$$+d_0[1+f_i(\eta_i-\eta_0)-\frac{\beta}{2}(\eta_i-\eta_0)^2+\frac{f_i^{'}}{6}(\eta_i-\eta_0)^3]$$
$$+c_0[f_i-\beta(\eta_i-\eta_0)+\frac{f_i^{'}}{2}(\eta_i-\eta_0)^2]$$
$$+b_0[-\beta+f_i^{''}(\eta_i-\eta_0)]$$
$$+a_0[f_i^{'}]=f_i^{'}f_i$$

Table – 1
Solution Of Problem Using Numerical Method

	Numerical solution with spline		
η	*S(η) for* $\beta=0$	*S(η) for* $\beta=1$	*S(η) for* $\beta=2$
0.0	*1.0000*	*1.0000*	*1.0000*
0.1	*1.0901*	*1.0840*	*1.0748*
0.2	*1.1617*	*1.1388*	*1.1033*
0.3	*1.2167*	*1.1678*	*1.0910*
0.4	*1.2571*	*1.1743*	*1.0420*
0.5	*1.2845*	*1.1607*	*0.9593*

A. Graphical Solution Of Given Problem:

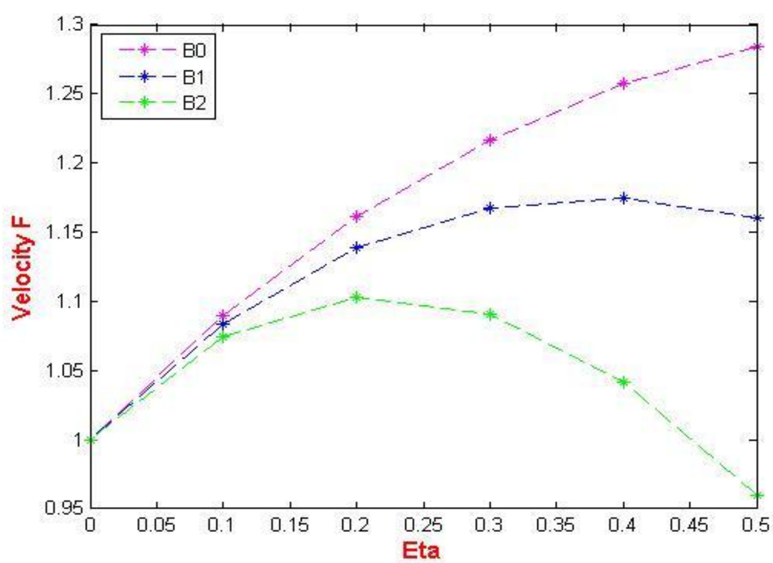

Fig. 1:

IV. RESULT AND DISCUSSION

The Graphical representation shows that, there is a significant impact of magnetic field on the velocity profile of the flow. Here we find velocity profile for different values of. From Figure it is clear that Value of increase the velocity of fluid decrease rapidly. It means that magnetic field increase, velocity of fluid flow decrease.

V. CONCLUSION

Solved the problem using quartic spline collocation method. The results are compared with the available results. This shows that spline method also gives nearest and accurate results. Also, the results are obtained one iterations only, which shows the reliability of the method. Thus we can solve such type of problems using numerical method

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