Flexure Analysis of Isotropic Skew Plate using MeshFree Method

Kumari Shipra Suman
M.E Student
Department of Mechanical Engineering
Birla Institute of Technology, Ranchi, 835215, India

Vikas
Assistant Professor
Department of Mechanical Engineering
Birla Institute of Technology, Ranchi, 835215, India

Jeeoot Singh
Associate Professor
Department of Mechanical Engineering
Birla Institute of Technology, Ranchi, 835215, India

Abstract

This paper presents the flexure analysis of a simply supported, thin to thick skew plate based on the higher-order shear deformation. Governing equation of the plate is obtained Hamilton's principle. A MATLAB code is developed incorporating polynomial radial basis function (RBF) base meshfree method to obtain the solution. Numerical results related to flexure analysis of skew plates are presented. Results are compared with the other published results to compare present solution methodology.

Keywords: Skew plate, Meshfree Method, Shear Deformation

I. INTRODUCTION

Plates are one of the important load carrying two dimensional structural elements being used in various high performance engineering structures ranging from deep in the ocean to high in the sky. Depending on the boundaries, a plate may be polygonal, quadrilateral, triangular, circular, elliptic etc. in shape. Rectangular, trapezoidal and skew plates are among the important members of the quadrilateral family that are used in aerospace, naval, automotive and other engineering structures. The analysis of skew plates as structural element gained the attention of researchers in the late 1940s. The exact solution for the structural response of skew plates is limited to specific cases. When the exact solution is not possible, the solution of partial differential equations can be obtained by analytical or numerical methods. In obtaining analytical solutions, the difficulty arises in forming the deflection function which can be applied to the entire plate domain that satisfies the boundary conditions. Numerical methods such as finite element, finite strip, differential quadrature, spline finite strip methods etc. are also employed for the analysis of skew plates. The skew plate problem has been widely obtained by finite element method (FEM). Sengupta (1995) has studied the performance of a simple finite element for the analysis of skew rhombic plates. The spline-finite-strip/element method has also been applied to the bending analysis of skew plates (Tham et al. 1986; Li et al. 1986; Wang and Hsu 1994). An exhaustive review on the work done on the bending analysis of skew plates before 1989 has been carried out by Butalia et al. (1990). Bending analysis of simply supported shear deformable Skew plates have been carried out by Liew and Han (1997).

II. MATHEMATICAL FORMULATION

The geometry of plate is shown in Figure-1. Thickness h is along z axis whose mid plane is coinciding with x-y plane of the coordinate system is considered.

![Fig. 1: Geometry of skew plate](image_url)

The displacement field at any point in the plate is expressed as:
\[ U = u_{ij}(x, y) - z \frac{\partial w_{ij}(x, y)}{\partial x} + \frac{h}{\pi} \sin \left( \frac{\pi z}{h} \right) \phi_i(x, y) \]

\[ V = v_{ij}(x, y) - z \frac{\partial w_{ij}(x, y)}{\partial y} + \frac{h}{\pi} \sin \left( \frac{\pi z}{h} \right) \phi_j(x, y) \]

\[ W = w_{ij}(x, y) \]

The governing differential equations of plate are obtained using Hamilton’s principle and expressed as:

\[ \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \]

\[ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = 0 \]

\[ \frac{\partial^2 M_{xx}}{\partial x^2} + \frac{\partial^2 M_{xy}}{\partial y^2} = q_x \]

\[ \frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{yy}}{\partial y} - Q_x = 0 \]

\[ \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yx}}{\partial y} - Q_y = 0 \]

The force and moment resultants in the plate and plate stiffness coefficients are expressed as:

\[ N_{ij}, M_{ij}, \sigma_{ij} = \int_{-h/2}^{+h/2} (\sigma_{ij}, z) \sigma_{ij}, \frac{h}{\pi} \sin \left( \frac{\pi z}{h} \right) \sigma_{ij} \, dz \]  

\[ Q_x, Q_y = \int_{-h/2}^{+h/2} (\sigma_{xx}, \sigma_{yy}) \left( \frac{h}{\pi} \right)^2 \cos \left( \frac{\pi z}{h} \right) \, dz \]

The boundary conditions for an arbitrary edge with simply supported conditions are as follows:

\[ u_{n_i}, v_{n_j}, \phi_{r_i}, w, M_{nn} = 0 \]

where

\[ u_{n_i} = n_{x_i} u + n_{y_i} v \]

\[ u_{n_j} = -n_{x_j} u + n_{y_j} v \]

\[ \phi_{r_i} = -n_{\phi_i} \phi + n_{\phi_j} \phi \]

\[ M_{nn} = n_{x_i} M_{xx} + 2n_{x_i} n_{y_i} M_{xy} + n_{y_i} M_{yy} \]

\[ n_{x} = \cos(\theta), \quad n_{y} = \sin(\theta) \]

**III. SOLUTION METHODOLOGY**

The governing differential equations (2-6) are expressed in terms of displacement functions. Radial basis function based formulation works on the principle of interpolation of scattered data over entire domain. A 2D rectangular domain having N\textsubscript{B} boundary nodes and N\textsubscript{D} interior nodes is shown in Figure-2.

The variable \(u, v, w, \phi\) can be interpolated in form of radial distance between nodes. The solution of the linear governing differential equations (4-8) is assumed in terms of polynomial radial basis function for nodes 1:N, as:

\[ u_{0}, v_{0}, w_{0}, \phi_{0}, \sigma_{0} = \sum_{j=1}^{N} (\alpha_{0}^{u}, \alpha_{0}^{v}, \alpha_{0}^{w}, \alpha_{0}^{\phi}, \alpha_{0}^{\sigma}) \cdot g \left( \|X - X_j\| \right) m \]

Where, N is total numbers of nodes which is equal to summation of boundary nodes NB and domain interior nodes ND. \(g \left( \|X - X_j\| \right) m \) is polynomial radial basis function expressed as \( g = r^m, \delta = \alpha_{j}^{u}, \alpha_{j}^{v}, \alpha_{j}^{w}, \alpha_{j}^{\phi}, \alpha_{j}^{\sigma} \) are unknown coefficients. \(\|X - X_j\|\) is the radial distance between two nodes.

Where, \( r = \|X - X_j\| = \sqrt{(x - x_j)^2 + (y - y_j)^2} \) and m is shape parameter. The value of ‘m’ taken here is 5.
Polynomial radial basis function becomes singular, when \( r = 0 \) i.e. for zero distance. In order to eliminate the singularity, an infinitesimally small value is added into the \( r^2 \) or zero distance. Mathematically it is explained as;
\[
r^2 = r^2 + \mu^2,
\]
when \( r = 0 \) or \( i = j \); \( \mu^2 \) is small numerical value of the order \( 10^{-10} \).

The discretized governing equations for linear flexural analysis can be written as:
\[
\begin{bmatrix}
[K]_L \\
[K]_B
\end{bmatrix}_{25N \times 5N}
\begin{bmatrix}
\{\delta\}
\end{bmatrix}_{5N \times 1} =
\begin{bmatrix}
\{F\}_L \\
0
\end{bmatrix}_{5N \times 1}
\]

(12)
The unknown coefficients \( \{\delta\} \) are calculated from equation (12) obtained and finally using equation (11), the displacement components at desired locations are obtained.

IV. NUMERICAL RESULTS AND DISCUSSIONS

In order to demonstrate the accuracy and applicability of present formulation, a RBF based meshless code in MATLAB is developed following the analysis procedure as discussed above. Several examples have been analyzed and the computed results are compared with the published results. Based on convergence study, a 13x13 node is used throughout the study. For isotropic material ( ) the deflection and moments are normalized as Liew and Han (1997).

\[
\begin{align*}
\bar{w} &= \frac{w_{\text{max}}}{1.600.D/(qa^4)} \\
M &= \frac{M_{\text{max}}}{40.D/(qa^4)} \\
\bar{\sigma}_{xx} &= \frac{\sigma_{xx \text{max}}}{h^2/(qa^4)} \\
\bar{\sigma}_{yy} &= \frac{\sigma_{yy \text{max}}}{h^2/(qa^4)}
\end{align*}
\]

(13)

here, subscript ‘c’ stands for center \((0,0,Z)\).

Fig. 2: An arbitrary two dimensional domains

Fig. 3: Convergence study for deflection of a simply supported skew plate \((a/h = 100)\)
Table - 1

<table>
<thead>
<tr>
<th>Skew Angle</th>
<th>90</th>
<th>75</th>
<th>60</th>
<th>45</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liew and Han (1997)</td>
<td>w</td>
<td>7.8469</td>
<td>7.0945</td>
<td>5.1714</td>
<td>2.8913</td>
</tr>
<tr>
<td>Present</td>
<td>w</td>
<td>7.7813</td>
<td>7.3379</td>
<td>6.0911</td>
<td>4.4017</td>
</tr>
<tr>
<td></td>
<td>(\sigma_{xx})</td>
<td>0.2941</td>
<td>0.2813</td>
<td>0.2441</td>
<td>0.1941</td>
</tr>
<tr>
<td></td>
<td>(\sigma_{xy})</td>
<td>0.2170</td>
<td>0.2250</td>
<td>0.1786</td>
<td>0.0886</td>
</tr>
<tr>
<td></td>
<td>(M_{nn})</td>
<td>1.9081</td>
<td>1.7438</td>
<td>1.3283</td>
<td>0.9900</td>
</tr>
</tbody>
</table>

Liew and Han (1997) | w | 1.9359 | 1.1546 | 0.3672 | 1.8151 | 2.2985 |
| Present | w | 6.8365 | 6.1390 | 4.3714 | 2.3191 | 0.7733 |
| | \(\sigma_{xx}\) | 6.7898 | 6.3137 | 5.0302 | 3.2893 | 1.8306 |
| | \(\sigma_{xy}\) | 0.2879 | 0.2729 | 0.3000 | 0.1650 | 0.1224 |
| | \(M_{nn}\) | 0.1984 | 0.2236 | 0.1914 | 0.0755 | 0.2507 |

Liew and Han (1997) | w | 1.9064 | 1.7346 | 1.3010 | 0.8158 | 0.6075 |
| Present | w | 1.6838 | 1.0998 | 0.2243 | 1.9063 | 1.5421 |
| | \(\sigma_{xx}\) | 6.5840 | 5.9001 | 4.1711 | 2.1751 | 0.6933 |
| | \(\sigma_{xy}\) | 6.5399 | 6.0431 | 4.7136 | 2.9172 | 1.4631 |
| | \(M_{nn}\) | 0.2864 | 0.2705 | 0.2252 | 0.1546 | 0.1028 |

Liew and Han (1997) | w | 0.1924 | 0.2284 | 0.1998 | 0.0789 | 0.1550 |
| Present | w | 1.9058 | 1.7279 | 1.2795 | 0.7726 | 0.4973 |
| | \(\sigma_{xx}\) | 1.5986 | 1.1071 | 0.2253 | 1.9124 | 1.2006 |
| | \(\sigma_{xy}\) | 6.5031 | 5.8236 | 4.1054 | 2.1204 | 0.6556 |
| | \(M_{nn}\) | 6.4598 | 5.9537 | 4.5992 | 2.7766 | 1.2931 |

Convergence for maximum deflection is shown in Fig 3. It can be seen that results are converge below one percent for 13x13 node. Hence for further analysis, a 13x13 node has been taken. Different results are obtained for deflection, moment and stresses and placed in Table-1 along with results of Liew and Han (1997). Present results are in good agreement. Fig 4 shows that as skew angle increases, the difference in present results becomes more.

![Fig. 4: Comparison of deflection of a simply supported skew plate (a/h = 100)](image-url)
The present study shows that the proposed RBFs are capable to accurately predict the flexure behavior of skew plates. Effect of skewness on deflection, moments and stresses is obtained. It is found that all the parameters decrease as skewness increases. Effect is more prominent for thick plates as compared to thin plate. The study further can be extended for orthotropic, laminated and FGM plates.

REFERENCES


Fig. 5: variation of Moment and stresses of a simply supported skew plate (a/h = 100)

Fig. 6: Effect of span to thickness ratio on deflection of a simply supported skew plate (a/h = 100)