Theory of Complex Fuzzy Soft Set and its Applications

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Abstract

The objective of this paper is to investigate the further development of theory of soft complex fuzzy set. Consequently, a major part of this work is dedicated to a discussion of the intuitive interpretation of aggregation operation in soft complex fuzzy set. We give an example of possible applications, which demonstrate the applications of aggregation operations that the method can be successfully applied to many problems that contains uncertainties and periodicities.
Keywords: complex fuzzy set, soft fuzzy set, soft complex fuzzy set, aggregations of soft complex fuzzy set

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I. INTRODUCTION

Complex fuzzy set (CFS) [11]-[12] is a new development in the theory of fuzzy systems [14]. The concept of CFS is an extension of fuzzy set, by which the membership for each element of a complex fuzzy set is extended to complex-valued state.

Soft set theory is a generalization of fuzzy set theory, which was proposed by Molodtsov [8] in 1999 to deal with uncertainty in a non-parametric manner. One of the most important steps for the theory of soft sets was to define mappings on soft sets; this was achieved in 2009 by mathematician Athar Kharal, though the results were published in 2011. Soft sets have also been applied to the problem of medical diagnosis for use in medical expert systems. Fuzzy soft sets have also been introduced in [10]. Mappings on fuzzy soft sets were defined and studied in the ground breaking work of Kharal and Ahmad.

Soft complex fuzzy sets , which is defined by [13], This paper written for inspired from [9,10], whereas all the concepts in soft sets were replaced by soft complex fuzzy sets.

The paper is organized as follows: Section 2 reviews the notions of soft sets; complex fuzzy set and relevant definitions used in the proposed work and also discussed the innovative concept of soft complex fuzzy sets with examples. In section 3, we introduce the aggregation operation on soft complex fuzzy set and its properties. In section 4, Applications of soft complex fuzzy set with example provided. We also demonstrate successful application of soft complex fuzzy set using aggregation operation. Finally we conclude the paper in section 5.

II. PRELIMINARIES

A. Definition 2.1.

Let U be an initial universe, P(U) be the power set of U, E be the set of all parameters and $A \subseteq E$. Then, a soft set as defined in [8] F_A over U is a set defined by a function f_A representing a mapping

$$f_A: E \rightarrow P(U) \text{ such that } f_A(x)=\phi \text{ if } x \notin A.$$

Here, f_A is called approximate function of the soft set F_A, and the value $f_A(x)$ is a set called x-element of the soft set for all $x \in E$. It is worth noting that the sets $f_A(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. Thus, a soft set F_A over U can be represented by the set of ordered pairs

$$F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\}$$

Note that the set of all soft sets over U will be denoted by S (U).

B. Definition 2.2. [8]

A pair (F, E) is called a soft set over U if and only if F is a mapping of E into the set of all subsets of the set U.

In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(\varepsilon)$, $\varepsilon \in$ E , from this family may be considered as the set of ε- elements of the soft set (F,E), or as the set of ε – approximate elements of the soft set.

C. *Example 2.3.*

Let U={h_1,h_2,h_3,h_4} be the set of four houses under consideration and E={p_1(costly),p_2(Beautiful),p_3(Modern Technology),p_4(luxurious),p_5(facility)}be the set of parameters and A={p_1,p_2,p_4}$\subseteq$ E. Then (F,A) = {F(p_1)={h_1,h_3}, F(p_2)={h_1,h_2,h_4},F(p_3={h_2}} is the soft set representing the 'attractiveness of the house' which a person is going to buy.

D. *Definition 2.4.*

Ramot et al. [10] proposed an important extension of these ideas, the *Complex Fuzzy Sets,* where the membership function μ instead of being a real valued function with the range [0,1] is replaced by a complex-valued function of the form

$$\mu_s(x) = r_s(x)e^{j\omega_s(x)}; \quad j=\sqrt{-1}$$

where $r_S(x)$ and $\omega_S(x)$ are both real valued giving the range as the unit circle. However, this concept is different from fuzzy complex number introduced and discussed by Buckley [2-5] and Zhang. Essentially as explained in [10,11] this still retains the characterization of the uncertainty through the amplitude of the grade of membership having a value in the range of [0, 1] whilst adding the membership phase captured by fuzzy sets. As explained in Ramot et al [10], the key feature of complex fuzzy sets is the presence of phase and its membership.

E. *Definition 2.5.*

Let *U* be an initial universe, *E* be the set of all parameters, $A \subseteq E$ and $\gamma_A(x)$ be a fuzzy set over *U* or all $x \in E$. Then, an Fuzzy Soft (*fs*-set) Γ_A over *U* is a set defined by a function γ_A representing a mapping,

$$\gamma_A : \text{E} \rightarrow \text{F(U) such that } \gamma_A(x) = \phi \text{ ; if } x \notin A.$$

Here, γ_A is called fuzzy approximate function of the *fs*-set Γ_A , and the value $\gamma_A(x)$ is a fuzzy set called *x*-element of the *fs*-set for all $x \in E$. Thus, an *fs*-set Γ_A over *U* can be represented by the set of ordered pairs,

$$\Gamma_A = \{(x, \gamma_A(x)) : x \in E, \gamma_A(x) \in F(U)\}$$

Note that the set of all the fuzzy sets over U will be denoted by F(U) and from now on the sets of all *fs*-sets over *U* will be denoted by *FS (U)*.

F. *Example 2.6.*

Let U={h_1,h_2,h_3,h_4} be the set of four houses under consideration and E={p_1(costly),p_2(Beautiful),p_3(Modern Technology), p_4(luxurious),p_5(facility)}be the set of parameters and A={p_1,p_2,p_4}$\subseteq$ E. Then (F,A)={F(p_1)={0.4/h_1,0.2/h_2,0.6/h_3,0.7/h_4}, F(p_2)={0.6/h_1,0.2/h_2,0.6/h_3,0.7/h_4},F(p_4)={0.4/h_1,0.3/h_2,0.7/h_3, 0.3/h_4}} is the fuzzy soft set representing the 'attractiveness of the house' which a person is going to buy.

G. *Definition 2.7.*

Let U be an initial universe, E be the set of all parameters, A$\subseteq$ E and $\psi_A(x)$ be a complex fuzzy set over U for all x$\in E$. Then, an Soft complex fuzzy set χ_A over U is a set defined by a function ψ_A representing a mapping

$$\psi_A : E \rightarrow C(U) : \text{such that } \psi_A(x) = \phi \text{ if } x \notin \text{A}.$$

Here, ψ_A is called complex fuzzy approximate function of the soft complex fuzzy set χ_A , and the value $\psi_A(x)$ is a complex fuzzy set called x-element of the soft complex fuzzy set for all x$\in$ E. Thus, a **soft complex fuzzy set** χ_A over U can be represented by the set of ordered pairs

$$\chi_A = \{(x, \psi_A(x)) : x \in E, \psi_A(x) \in C(U)\}$$

Note that the set of all the complex fuzzy sets over U will be denoted by C(U) . Operations of complex fuzzy sets and soft complex fuzzy sets were defined in [1,13] respectively.

H. *Example 2.8.*

[t] Let U= {h_1 (India),h_2(Australia),h_3(UK),h_4(USA)} be an initial set, consider E= {e_1(Inflation rate),e_2(population growth),e_3(Unemployment rate),e_4(share market index)} be an country's growth parameters set and A$\subseteq$ E,A={e_1,e_3}, complex fuzzy set $\psi_A(e_1)$, $\psi_A(e_3)$ is defined as,

$$\psi_A(e_1) = \left\{ \frac{0.4e^{j0.5\pi}}{h_1}, \frac{0.8e^{j0.6\pi}}{h_2}, \frac{0.8e^{j0.8\pi}}{h_3}, \frac{1.0e^{j0.75\pi}}{h_4} \right\},$$

$$\psi_A(e_3) = \left\{ \frac{0.6e^{j0.7\pi}}{h_1}, \frac{0.9e^{j0.9\pi}}{h_2}, \frac{0.7e^{j0.95\pi}}{h_3}, \frac{0.75^{j0.95\pi}}{h_4} \right\}$$

then soft complex fuzzy set χ_A is written by,

$$\chi_A = \left\{ \left(e_1, \frac{0.4e^{j0.5\pi}}{h_1}, \frac{0.8e^{j0.6\pi}}{h_2}, \frac{0.8e^{j0.8\pi}}{h_3}, \frac{1.0e^{j0.75\pi}}{h_4} \right), \left(e_3, \frac{0.6e^{j0.7\pi}}{h_1}, \frac{0.9e^{j0.9\pi}}{h_2}, \frac{0.7e^{j0.95\pi}}{h_3}, \frac{0.75^{j0.95\pi}}{h_4} \right) \right\}$$

III. AGGREGATION OF SOFT COMPLEX FUZZY SET

In this section, we define an aggregation operator on soft complex fuzzy set that produces an aggregate fuzzy set from a soft complex fuzzy set and its cardinal set. The approximate functions of a soft complex fuzzy set are fuzzy. A soft complex fuzzy set aggregation operator on the fuzzy sets is an operation by which several approximate functions of a soft complex fuzzy set are combined to produce a single fuzzy set which is the aggregate fuzzy set of the soft complex fuzzy set. Once an aggregate fuzzy set has been arrived at, it may be necessary to choose the best single crisp alternative from this set.

A. Definition 3.1.

Let $\chi_A \in C(U)$. Assume that $U = \{u_1, u_2, ..., u_m\}$ $E = \{x_1, x_2, ..., x_n\}$ and $A \subseteq E$, then the χ_A can be presented by the following table

χ_A	x_1	x_2	$\dots$	x_n
u_1	$\mu_{\psi_A(x_1)}(u_1)$	$\mu_{\psi_A(x_2)}(u_1)$	$\dots$	$\mu_{\psi_A(x_n)}(u_1)$
u_2	$\mu_{\psi_A(x_1)}(u_2)$	$\mu_{\psi_A(x_2)}(u_2)$	$\dots$	$\mu_{\psi_A(x_n)}(u_2)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
u_m	$\mu_{\psi_A(x_1)}(u_m)$	$\mu_{\psi_A(x_2)}(u_m)$	$\dots$	$\mu_{\psi_A(x_n)}(u_m)$

Where $\mu_{\psi_A(x)}$ is the membership function of ψ_A.

If $a_{ij} = \mu_{\psi_A(x_1)}(u_i)$, for $i = 1,2,..., m$ and $j = 1,2,..., n$, then the soft complex fuzzy set χ_A is uniquely characterized by a matrix, $[a_{ij}]_{mXn} \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$ is called an $m \times n$ soft complex fuzzy set matrix of the soft complex fuzzy set χ_A over U.

B. Definition 3.2.

Let $\chi_A \in C(U)$. Then, the cardinal set of χ_A, denoted by ${}_C\chi_A$ is defined by ${}_C\chi_A = \{\mu_{{}_C\chi_A}(x)/x : x \in E\}$, is a fuzzy set over E. The membership function $\mu_{{}_C\chi_A}$ of ${}_C\chi_A$ is defined by $\mu_{{}_C\chi_A} : E \rightarrow [0,1]$, $\mu_{{}_C\chi_A} = \frac{|\psi_A(x)|}{|U|}$

where $|U|$ is the cardinality of universe U, and $|\psi_A(x)|$ is the scalar cardinality of fuzzy set $\psi_A(x)$. Note that the set of all cardinal sets of the soft complex fuzzy sets over U will be denoted by ${}_C C(U)$. It is clear that ${}_C C(U) \subseteq F(E)$

C. Definition 3.3.

Let $\chi_A \in C(U)$ and ${}_C\chi_A \in {}_CC(U)$. Assume that $E = \{x_1, x_2, ..., x_n\}$ and $A \subseteq E$, then ${}_C\chi_A$ can be presented by the following table

E	x_1	x_2	$\cdots$	x_n
$\mu_C\chi_A$	$\mu_{C\chi_A}(x_1)$	$\mu_{C\chi_A}(x_2)$	$\cdots$	$\mu_{C\chi_A}(x_n)$

If $a_{1j} = \mu_{C\chi_A}(x_j)$ for j=1,2,…,n, then the cardinal set ${}_C\chi_A$ is uniquely characterized by a matrix, $[a_{1j}]_{1\times n} = [a_{11}\ a_{12}\ ...a_{1n}]$ which is called the cardinal matrix of the cardinal set ${}_C\chi_A$ over E.

D. Definition 3.4.

Let $\chi_A \in C(U)$ and ${}_C\chi_A \in {}_CC(U)$. Then soft complex fuzzy aggregation operator, denoted by CFS_{agg}, is defined by

$CFS_{agg} : {}_CC(U) \times C(U) \to F(U), CFS_{agg}({}_C\chi_A, \chi_A) = \chi_A^*$

Where $\chi_A^* = \{\mu_{\chi_A^*}(u)/u : u \in U\}$ a fuzzy set is over U. χ_A^* is called the aggregate fuzzy set of the soft complex fuzzy set χ_A. The membership function $\mu\chi_A^*$ of χ_A^* is defined as follows:

$\mu\chi_A^* : U \to [0,1],\ \mu\chi_A^*(u) = \frac{1}{|E|}\sum_{x\in E}\mu_{C\chi_A}(x)\mu_{C\psi_A}(u)$, where |E| is the cardinality of E.

E. Definition 3.5.

Let $\chi_A \in C(U)$ and χ_A^* be its aggregate fuzzy set. Assume that $U = \{u_1, u_2, ..., u_m\}$, then the χ_A^* can be presented by the following table

χ_A	$\mu\chi_A^*$
u_1	$\mu\chi_A^*(u_1)$
u_2	$\mu\chi_A^*(u_2)$
$\vdots$	$\vdots$
u_m	$\mu\chi_A^*(u_m)$

If $a_{i1} = \mu\chi_A^*(u_i)$ for i=1,2,…,m, then χ_A^* is uniquely characterized by the matrix, $[a_{i1}]_{m\times 1} = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$ which is called the aggregate matrix of χ_A^* over U.

IV. APPLICATIONS OF SOFT COMPLEX FUZZY SET

It is much easier to describe a human behavior directly showing the set of strategies which a person may choose in a particular situation especially a periodic phenomenon. The situation may be more complicated in real world because of the fuzzy characters of the parameters [9, 10]. In fuzzy set, the complex fuzzy set is extended to a fuzzy one; the complex fuzzy membership is used to describe the parameter approximate elements of soft complex fuzzy set.

This theory also used in data mining applications and decision making problem. This theory used in study of smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, probability, Theory of measurement, etc.

Once an aggregate fuzzy set has been arrived at, it may be necessary to choose the best alternative from this set. Therefore, we can make a decision by the following algorithm.

1) Step 1: Construct an soft complex fuzzy set χ_A over U,
2) Step 2: Find the cardinal set ${}_C\chi_A$ of χ_A for amplitude term and phase term in seperately,
3) Step 3: Find the aggregate fuzzy set χ_A^* of χ_A also find for seperately ,

4) Step 4: Find the best alternative from this set that has the largest membership grade by max modulus of $\mu\chi_A^*(u)$

F. *Example 4.1.*

Suppose a business man want to buy a share from share market. There are three same kind of share which form the set of alternatives, $U = \{u_1, u_2, u_3, u_4\}$. The expert committee consider a set of parameters, $E = \{x_1, x_2, x_3, x_4\}$. For i = 1, 2, 3, 4, the parameters x_i stand for "current trend of company performance", "particular company's stock price for last one year", " Home country inflation rate", and "current situation of particular share's country share market", respectively. $A = \{x_1, x_2, x_3\}$, and $A \subseteq E$, complex fuzzy sets $\psi_A(x_1)$, $\psi_A(x_2)$, and $\psi_A(x_3)$ is defined as,

$$\psi_A(x_1) = \left\{\frac{0.4e^{j0.5\pi}}{u_1}, \frac{0.8e^{j0.6\pi}}{u_2}, \frac{0.8e^{j0.8\pi}}{u_3}, \frac{1.0e^{j0.75\pi}}{u_4}\right\}, \quad \psi_A(x_2) = \left\{\frac{0.3e^{j0.7\pi}}{u_1}, \frac{0.6e^{j0.8\pi}}{u_2}, \frac{0.5e^{j0.2\pi}}{u_3}, \frac{1.0e^{j0.85\pi}}{u_4}\right\},$$

$$\text{And } \psi_A(x_3) = \left\{\frac{0.6e^{j0.7\pi}}{u_1}, \frac{0.9e^{j0.9\pi}}{u_2}, \frac{0.7e^{j0.95\pi}}{u_3}, \frac{0.75^{j0.95\pi}}{u_4}\right\}$$

Then,

1) Step.1: Soft complex fuzzy set χ_A is written by,

$$\chi_A = \left\{ \begin{array}{l} \left(x_1, \frac{0.4e^{j0.5\pi}}{u_1}, \frac{0.8e^{j0.6\pi}}{u_2}, \frac{0.8e^{j0.8\pi}}{u_3}, \frac{1.0e^{j0.75\pi}}{u_4}\right), \left(x_2, \frac{0.3e^{j0.7\pi}}{u_1}, \frac{0.6e^{j0.8\pi}}{u_2}, \frac{0.5e^{j0.2\pi}}{u_3}, \frac{1.0e^{j0.85\pi}}{u_4}\right), \\ \left(x_3, \frac{0.6e^{j0.7\pi}}{u_1}, \frac{0.9e^{j0.9\pi}}{u_2}, \frac{0.7e^{j0.95\pi}}{u_3}, \frac{0.75^{j0.95\pi}}{u_4}\right) \end{array} \right\}$$

2) Step.2: The cardinal is computed,

$$_C\chi_A(Amplitude\ \ Term) = \{0.75/x_1, 0.6/x_2, 0.74/x_3\}$$

$$_C\chi_A(Phase\ \ Term) = \{0.66/x_1, 0.64/x_2, 0.87/x_3\}$$

3) Step.3: The aggregate fuzzy set is found by following method,

$$M_{\chi_A^*}(Amplitude\ \ Term) = \frac{1}{4}\begin{bmatrix} 0.4 & 0.3 & 0.6 & 0 \\ 0.8 & 0.6 & 0.9 & 0 \\ 0.8 & 0.5 & 0.7 & 0 \\ 1.0 & 1.0 & 0.75 & 0 \end{bmatrix}\begin{bmatrix} 0.75 \\ 0.6 \\ 0.74 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.23 \\ 0.41 \\ 0.35 \\ 0.47 \end{bmatrix}$$

$$M_{\chi_A^*}(Phase\ \ Term) = \frac{1}{4}\begin{bmatrix} 0.5 & 0.7 & 0.6 & 0 \\ 0.6 & 0.8 & 0.9 & 0 \\ 0.8 & 0.2 & 0.7 & 0 \\ 0.75 & 0.85 & 0.75 & 0 \end{bmatrix}\begin{bmatrix} 0.66 \\ 0.64 \\ 0.87 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.33 \\ 0.42 \\ 0.32 \\ 0.42 \end{bmatrix}$$

$$\chi_A^* = \{0.23e^{0.33\pi}/u_1, 0.41e^{0.42\pi}/u_2, 0.35e^{0.32\pi}/u_3, 0.47e^{0.42\pi}/u_4\}$$

Consider the modulus value of $Max(\mu\chi_A^*) = \{0.4/u_1, 0.58/u_2, 0.47/u_3, 0.63/u_4\} = 0.63/u4$

This means that the 4th share {u4} has the largest membership grade. Hence expert committee may be suggested for buy 4th share among other shares.

V. Conclusion

In soft complex fuzzy sets, the soft set theory is extended to a complex fuzzy set; the fuzzy membership is used to describe parameter approximate elements of complex fuzzy soft set. To develop the theory, in this work, we introduced aggregate operation of soft complex fuzzy set and its application in decision making problems. Finally, we provided an example demonstrating the successfully application of this method. It may be applied to many fields with problems that contain uncertainty and periodicity, and it would be beneficial to extend the proposed method to subsequent studies in complex fuzzy set.

Thus in all, the soft complex fuzzy set seems to be promising new concept, paving the way to numerous possibilities for future research.

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