The Split and Non Split Majority Domonation in Fuzzy Graphs

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Abstract

A majority dominating set D of a fuzzy graph G is a split majority dominating set if the induced fuzzy sub graph \( \langle V - D \rangle \) is disconnected. A majority dominating set D of a fuzzy graph G is a non-split majority dominating set if induced fuzzy sub graph \( \langle V - D \rangle \) is connected. In this paper we study split and non-split majority domination in fuzzy graphs and its domination numbers \( \gamma_{SM}(G) \) and \( \gamma_{NSM}(G) \). Also bounds \( \gamma_{SM}(G) \) and \( \gamma_{NSM}(G) \) with other known parameters are discussed.

Keywords: Dominating set, Majority dominating set, split majority dominating set, non-split majority dominating set

I. INTRODUCTION

A subset \( D \subseteq V \) in a fuzzy graph G is called a majority dominating set if atleast half of the vertices of of G are either in D or adjacent to the vertices of D. More clearly \( |N(D)| \geq \left\lceil \frac{p}{2} \right\rceil \)

A majority dominating set D is minimal if no proper subset of D is a majority dominating set. The minimum fuzzy cardinality of a minimal majority dominating set is called the majority domination number and it is denoted by \( \gamma_{M}(G) \)

The split majority domination number \( \gamma_{SM}(G) \) of G is the minimum fuzzy cardinality of a minimal split majority dominating set.

A set D of vertices in a fuzzy graph G is dominating set if every vertex \( v \in V \) is either an element of D or adjacent to an element of D. A dominating set is called minimal dominating set if no proper subset of D is a dominating set. The minimum fuzzy cardinality of a minimal dominating set is called the domination number of a fuzzy graph G and it is denoted by \( \gamma(G) \)

A set D of vertices of a fuzzy graph G is said to be majority independent set if it induces a totally disconnected sub graph with \( |N(D)| \geq \left\lceil \frac{p}{2} \right\rceil \) \( \forall v \in D \).

If any vertex \( D' \) properly containing D is not majority independent set, then D is called maximal majority independent set. The maximum fuzzy cardinality of a maximal majority independent set is called majority independent number and it is denoted by \( \beta_{M}(G) \)

Example:
A. **Theorem:**

A majority dominating set D of a fuzzy graph G is split majority dominating set iff there exists two vertices w₁ and w₂ from two components of V-D such that w₁ - w₂ path contains a vertex of D.

**Proof:**

Suppose D is a split majority dominating set of G. Then \( |V - D| \) is disconnected and it must contain at least two components G₁ and G₂.

Let \( w₁ \in G₁ \) and \( w₂ \in G₂ \). Now \( w₁ - w₂ \) would be a path through a vertex \( v \in D \). This path contains a vertex \( u \) of D. Conversely, let D be a majority dominating set such that V-D is disconnected. This implies that D is a split majority dominating set of G.

B. **Theorem:**

If a fuzzy graph G has one cut vertex \( v \) and at least two blocks \( H₁ \) and \( H₂ \) with \( v \) adjacent to all vertices of \( H₁ \) and \( H₂ \), then \( v \) is in every \( SM \gamma \) set of G.

**Proof:**

Let \( D \) be a \( SM \gamma \) set of G. Then \( |V - D| \) is disconnected. This implies that \( D = \{ u, w \} \) is a split majority dominating set of G. This contradicts that \( v \) is adjacent to all vertices of \( H₁ \) and \( H₂ \). Hence, \( v \) is in every \( SM \gamma \) set of G.

C. **Theorem:**

For a fuzzy graph G

1) \( \kappa(G) \leq \gamma_{SM}(G) \leq \gamma(S)(G) \) where \( \kappa \) is vertex connectivity.

2) \( \gamma_M(G) \leq \gamma_{SM}(G) \)

3) \( \gamma_M(G) \leq \gamma_{SM}(G) \leq \gamma(G) \leq \gamma(S)(G) \)

**Proof:**

Let \( D \) be a \( \gamma \) set of a fuzzy graph G. Then \( D \) is also a split majority dominating set of G. Therefore \( \gamma_{SM}(G) \leq |D| = \gamma(S)(G) \)

If \( S \) is a \( SM \gamma \) set of G. Then \( |V - S| \) is disconnected.

Therefore, the minimum number of vertices in \( S \) would disconnect G and hence

\[ \kappa(G) \leq |S| = \gamma_{SM}(G) \]

1) Since every split majority dominating set \( S \) of G is a majority dominating set of G,

\[ \gamma_M(G) \leq |S| = \gamma_{SM}(G) \]

2) Since \( \gamma_{SM}(G) \leq \gamma(G) \), \( \gamma_M(G) \leq \gamma_{SM}(G) \) \( \gamma(G) \leq \gamma(S)(G) \) we have

\[ \gamma_M(G) \leq \gamma_{SM}(G) \leq \gamma(G) \leq \gamma(S)(G) \]
**D. Theorem:**

A fuzzy tree $T$ has a majority dominating vertex adjacent to more than one pendent vertex or $T$ has a support vertex iff every

$\gamma_M$ set of $T$ is also a $\gamma_{SM}$ set of $T$

1) **Proof:**

Let $D$ be a $\gamma_M$ set of a fuzzy tree $T$. Assume that $T$ has a majority dominating set $v$ adjacent to more than one pendent vertex.

Then $v$ must be in $D$ and so $D$ is a $\gamma_{SM}$ set of $T$.

Suppose $v$ is not a supporting vertex in $T$. Then $D$ contains either $v$ or atleast one support adjacent to $v$ or a nonsupport adjacent to $v$. In this case $\langle V - D \rangle$ is disconnected and so $D$ is a $\gamma_{SM}$ set of $T$.

Conversely, suppose every $\gamma_M$ set $D$ of $T$ is also a $\gamma_{SM}$ set of $T$. Then every $\gamma_M$ set $D$ of $T$ is also a $\gamma_{SM}$ set of $T$. Then every $\langle V - D \rangle$ is disconnected.

Case i) Suppose both minimal majority dominating set and minimal split majority dominating set contains only one vertex. Then $T$ has a majority dominating vertex $v$ and $D = \{v\}$ since the is disconnected, $v$ is adjacent to more than one pendent vertex.

Case ii) Suppose $\gamma_M$ set contains 2 or more vertices, then $T$ has no majority dominating vertex $v$. So, we have the following cases:

1) $D$ contains only supports
2) $D$ contains only non support vertices
3) $D$ contains non support and support vertices

**E. Theorem:**

For a fuzzy tree $T$ every $\gamma_{NSM}$ set contains at least one end vertex.

1) **Proof:**

Let $D$ be a $\gamma_{NSM}$ of a fuzzy tree $T$. Suppose $D$ does not contain any end vertex $v$. Then $D$ contains either support or intermediate vertices. If $D$ contains only supports or only intermediate vertices or both supports and intermediate vertices, every vertex will be a cut vertex of $G$ and $\langle V - D \rangle$ is disconnected.

Therefore, every $\gamma_{NSM}$ set contains at least one end vertex.

**F. Theorem:**

Let $G = G_1$ and $G_2$ be fuzzy graphs. If the join $G = G_1 + G_2$, then $\gamma_{NSM}(G) \leq 1$

1) **Proof:**

Let $G = G_1 + G_2$ and $\langle V(G) \rangle = \langle V(G_1) \rangle \cup \langle V(G_2) \rangle \equiv p$, then by definition of a join of two fuzzy graphs $G_1$ and $G_2$, every vertex in $G_1$ is adjacent to every vertex in $G_2$.

Suppose $|G_1| = |G_2|$. Then $N(v) \geq \left\lceil \frac{p}{2} \right\rceil \forall v \in G_1 \cup G_2$.

Since $\delta(G^*) > 1$, $\langle V - D \rangle$ is connected.

Therefore $\gamma_{NSM}(G) \leq 1$. If $|G_1| < \left\lceil \frac{p}{2} \right\rceil$

Let $|G_1| < \left\lceil \frac{p}{2} \right\rceil$. Then $|G_2| \geq \left\lceil \frac{p}{2} \right\rceil$.

Since every vertex in $G_1$ is adjacent to every vertex in $G_2$, $N(v) > \left\lceil \frac{p}{2} \right\rceil \forall v \in G_1$ and $\langle V - D \rangle$ is connected. In the same way we prove the theorem $|G_2| < \left\lceil \frac{p}{2} \right\rceil$.

Therefore $\gamma_{NSM}(G) \leq 1$.
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REFERENCES


