The Split and Non Split Majority Domonation in Fuzzy Graphs

DR. C. V. R. Harinarayanan
Research Supervisor & Assistant Professor
Government Arts College, Paramakudi.

S.Geetha
Assistant Professor
Kings College of Engineering, Punalkulam.

Dr.R.Muthuraj
Research Supervisor & Assistant Professor
H.H.The Rajah's College(Autonomous), Pudukkottai

Abstract

A majority dominating set D of a fuzzy graph G is a split majority dominating set if the induced fuzzy sub graph $\langle V-D\rangle$ is disconnected. A majority dominating set D of a fuzzy graph G is a non- split majority dominating set if induced fuzzy sub graph $\langle V-D\rangle$ is connected. In this paper we study split and non-split majority domination in fuzzy graphs and its domination numbers $\gamma_{SM}(G)$ *and* $\gamma_{NSM}(G)$ .Also bounds $\gamma_{SM}(G)$ *and* $\gamma_{NSM}(G)$ with other known parameters are discussed.

Keywords: Dominating set, Majority dominating set, split majority dominating set, non-split majority dominating set

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I. INTRODUCTION

A subset D $\subseteq V$ in a fuzzy graph G is called a majority dominating set if atleast half of the vertices of of G are either in D or adjacent to the vertices of D. More clearly $|N(D)| \geq \left\lceil \frac{p}{2} \right\rceil$

A majority dominating set D is minimal if no proper subset of D is a majority dominating set . The minimum fuzzy cardinality of a minimal majority dominating set is called the majority domination number and it is denoted by $\gamma_M(G)$

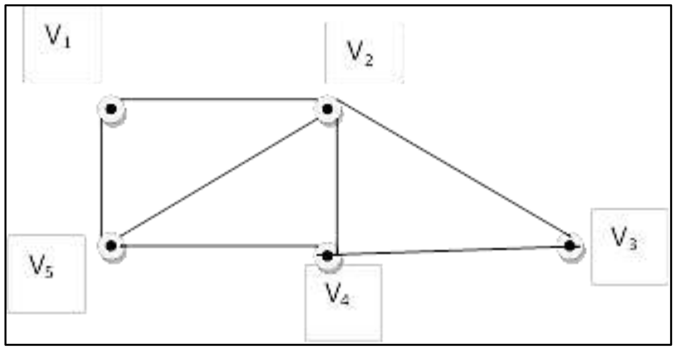
The split majority domination number $\gamma_{SM}(G)$ of G is the minimum fuzzy cardinality of a minimal split majority dominating set.

A set D of vertices in a fuzzy graph G is dominating set if every vertex $v \in$ V is either an element of D or adjacent to an element of D. A dominating set is called minimal dominating set if no proper subset of D is a dominating set. The minimum fuzzy cardinality of a minimal dominating set is called the domination number of a fuzzy graph G and it is denoted by $\gamma(G)$

A set D of vertices of a fuzzy graph G is said to be majority independent set if it induces a totally disconnected sub graph with $|N(D)| \geq \left\lceil \frac{p}{2} \right\rceil \forall v \in D.$

If any vertex D' properly containing D is not majority independent set, then D is called maximal majority independent set. The maximum fuzzy cardinality of a maximal majority independent set is called majority independent number and it is denoted by $\beta_M(G)$

Example:



$\sigma(v_1) = 0.1 \qquad \sigma(v_2) = 0.2 \qquad \sigma(v_3) = 0.3 \qquad \sigma(v_4) = 0.2 \qquad \sigma(v_5) = 0.5$

$D = \{v_2\}$ *is a dominating set*

Majority dominating set is $= \{v_1, v_3, v_5\}$

A. Theorem:

A majority dominating set D of a fuzzy graph G is split majority dominating set iff there exists two vertices w_1 and w_2 from two components of V-D such that w_1 -w_2 path contains a vertex of D

1) Proof:

Suppose D is a split majority dominating set of G. Then $\langle V - D \rangle$ is disconnected and it must contain atleast two components G_1 and G_2

Let $w_1 \in G_1$ *and* $w_2 \in G_2$. Now w_1 -w_2 would be a path through a vertex v $\in$ D. This path contains a vertex u of D. Conversely, let D be a majority dominating set such that V-D is disconnected. This implies that D is a split majority dominating set of G.

B. Theorem:

If a fuzzy graph G has one cut vertex v and at least two blocks H_1 and H_2 with v adjacent to all vertices of H_1 and H_2, then v is in every γ_{SM} set of G

1) Proof:

Let D be a γ_{SM} set of G

Suppose v$\in$ V-D. Then each of H_1 and H_2 contributes atleast one vertex to D say u and w respectively.

This implies that $D - \{u, w\}$ is a split majority dominating set of G. This contradicts that v is adjacent to all vertices of H_1 and H_2. Hence, v is in every γ_{SM} set of G.

C. Theorem:

For a fuzzy graph G

1) $\kappa(G) \leq \gamma_{SM}(G) \leq \gamma_S(G)$ *where* κ *is vertexconn ectivity*
2) $\gamma_M(G) \leq \gamma_{SM}(G)$
3) $\gamma_M(G) \leq \gamma_{SM}(G) \leq \gamma(G) \leq \gamma_S(G)$

1) Proof:

Let D be a γ_S - set of a fuzzy graph G. Then D is also a split majority dominating set of G. Therefore $\gamma_{SM}(G) \leq |D| = \gamma_S(G)$

If S is a γ_{SM} - set of G. Then $\langle V - S \rangle$ is disconnected.

Therefore, the minimum number of vertices in S would disconnect G and hence

$$\kappa(G) \leq |S| = \gamma_{SM}(G)$$

1) Since every split majority dominating set S of G is a majority dominating set of G,

$$\gamma_M(G) \leq |S| = \gamma_{SM}(G)$$

2) Since $\gamma_{SM}(G) \leq \gamma(G)$, $\gamma_M(G) \leq \gamma_{SM}(G)$ and $\gamma(G) \leq \gamma_S(G)$ we have $\gamma_M(G) \leq \gamma_{SM}(G) \leq \gamma(G) \leq \gamma_S(G)$

D. *Theorem:*

A fuzzy tree T has a majority dominating vertex adjacent to more than one pendent vertex or T has a support vertex iff every γ_M set of T is also a γ_{SM} set of T

1) *Proof:*

Let D be a γ_M set of a fuzzy tree T . Assume that T has a majority dominating set v adjacent to more than one pendent vertex . Then v must be in D and so D is a γ_{SM} set of T

Suppose v is not a supporting vertex in T. Then D contains either v or atleast one support adjacent to v or a nonsupport adjacent to v. In this case $\langle V-D \rangle$ is disconnected and so D is a γ_{SM} set of T

Conversely, suppose every γ_M set D of T is also a γ_{SM} set of T. Then every γ_M set D of T is also a γ_{SM} set of T. Then every $\langle V-D \rangle$ is disconnected.

Case i) Suppose both minimal majority dominating set and minimal split majority dominating set contains only one vertex. Then T has a majority dominating vertex v and D= $\langle v \rangle$ .Since the is disconnected, v is adjacent to more than one pendent vertex.

Case ii) Suppose γ_M set γ_{SM} set contains 2 or more vertices, then T has no majority dominating vertex v. So, we have the following cases:

1) D contains only supports
2) D contains only non support vertices
3) D contains non support and support vertices

E. *Theorem:*

For a fuzzy tree ,T every γ_{NSM} -set contains atleast one end vertex

1) *Proof:*

Let D be a γ_{NSM} - set of a fuzzy tree . Suppose D does not contain any end vertex v. Then D contains either support or intermediate vertices. If D contains only supports or only intermediate vertices or both supports and intermediate vertices , every vertex will be a cut vertex of G and $\langle V-D \rangle$ is disconnected.

Therefore , every γ_{NSM} -set contains atleast one end veretex.

F. *Theorem:*

Let G = G_1 and G_2 be fuzzy graphs. If the join G= G_1+ G_2, then $\gamma_{NSM}(G) \leq 1$

1) *Proof:*

Let G== G_1+ G_2 and $|V(G) = |V(G_1)| + |V(G_2)|$ =p , then by definition of join of two fuzzy graphs G_1 and G_2, every vertex in G_1 is adjacent to every vertex in G_2.

Suppose $|G_1| = |G_2|$. Then $\left| N(v) \geq \left\lceil \frac{p}{2} \right\rceil \right| \forall v \in G_1 \cup G_2$

Since $\delta(G^*) > 1, \langle V-D \rangle$ is connected.

Therefore $\gamma_{NSM}(G) \leq 1$. If $|G_1|$ *or* $|G_2| < \left\lceil \frac{p}{2} \right\rceil$

Let $|G_1| < \left\lceil \frac{p}{2} \right\rceil$. *Then* $|G_2| \geq \left\lceil \frac{p}{2} \right\rceil$

Since every vertex in G_1 is adjacent to every vertex in G_2, $\left| N(v) > \left\lceil \frac{p}{2} \right\rceil \right| \forall v \in G_1$ and $\langle V-D \rangle$ is connected. In the same way we prove the theorem $|G_2| < \left\lceil \frac{p}{2} \right\rceil$

Therefore $\gamma_{NSM}(G) \leq 1$

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