Optimal Operating Policies of EPQ Model for Deteriorating Items with Time Dependent Production having Production Quantity Dependent Demand

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Abstract

Inventory models play an important role in determining the optimal ordering policies. Much work has been reported in literature regarding inventory models with finite or infinite replenishment. But in many practical situations the replenishment is governed by random factors like procurement, transportation, environmental condition, availability of raw material etc., hence, it is needed to develop inventory models with different types of demand. In this paper we develop and analyze an inventory model with the assumption that the demand is power pattern and follows an exponential distribution. However, in some other inventory systems the demand is dependent on production quantity for example in oil exploration industries, manufacturing industries; the demand is a function of production quantity. To analyze this sort of situations an economic production quantity (EPQ) model for deteriorating item is developed with the assumption that the production rate is time dependent and demand is dependent on the production quantity Q. It is further assumed that the demand rate is of the form $\lambda(t) = \tau + \eta Q^{v}, \quad 0 \le t \le T, \ 0 \le \eta \le 1, \tau > 0, v > 0$ where η, λ and τ are positive constants. This demand rate also includes the constant rate of demand, when $\eta = 0$. The instantaneous level of inventory at any given time't' is derived through differential equations. With suitable cost considerations the optimal ordering policies are obtained.
Keywords: EPQ model, Demand, Production rate, Production quantity dependent demand, Production scheduling

I. INTRODUCTION

Inventory models create a lot of interest due to their ready applicability at various places such as production processes, manufacturing areas, business, administration, warehousing, supply chain management, and market yards and so on. Mathematical models provide the basic frame work for the analysis of many practical situations. In the same field Frade Henson defined inventory model as the stock of goods kept for future use. Even though the inventories are essential and they provide an alternative to production or purchase in future, they also mean locking up of capital of an enterprise. Maintance of inventories also cost much by way of expenses on stores, equipments, personal insurance etc,. Thus in the process excess inventories become undesirable. Hence, inventory control and management play a dominant role in the last five decades and much work is reported in literature regarding inventory models with various assumptions. Inventory models are classified under two categories, such as inventory models for deteriorating items and inventory models for non-deteriorating items. They are based according to the life time of the commodity, either finite or infinite. Deteriorating items refer to the items that become decayed, damaged, evaporative, expired, invalid, devalued and so on through time (Wee IIM (1993)). Deteriorating items can be classified into two categories. The first category refers to the items that become decayed, damaged, evaporated, or expired through time such as meat, vegetables, fruit, medicine, flowers, film, etc,. The other category refers to the items that lose part or total value through time, like computer chips, mobile phones, fashion related items and seasonal goods, etc,.

In classical economic production quantity (EPQ) models, it is customary to assume that the production rate is pre-determined and inflexible. But in many practical situations, the production rate is dependent on several factors like availability of raw material, skill levels of the employees, machine life, quality requirements, etc,. Hence, the production rate cannot be considered as constant throughout the period of production cycle. An economic production quantity (EPQ) model for deteriorating items with demand as a function of production quantity and time dependent production rate is developed and analyzed with shortages. Then the optimal operating policies of the model are derived. The sensitivity of the model with respect to cost and parameters is also presented. However, in some other inventory systems the demand is dependent on production quantity, for example in oil exploration industries, manufacturing industries; where the demand is a function of production quantity. To analyze these sorts of situations, an economic production quantity (EPQ) model for deteriorating item is developed with the assumption that the production rate is time dependent and demand is dependent on the production quantity Q. It is further assumed that the demand rate is of the form $\lambda(t) = \tau + \eta Q^{v}, \quad 0 \le t \le T, \ 0 \le \eta \le 1, \tau > 0, v > 0$ where η, λ and τ are positive constants. This demand rate also includes the constant rate of demand, when $\eta = 0$.Using differential equations, the instantaneous state of inventory is derived. By minimizing the total cost function, the optimal production quantity, production down time and production up time are derived with shortages. The sensitivity of the model with respect to the parameters and costs is studied.

II. NOTATIONS

The following notations are used for developing the model.

- : Deterioration parameter.
- Q: Ordering quantity in one cycle
- A: Ordering cost
- C: Cost per unit
- R: Total production
- n: Index parameter
- h: Inventory holding cost per unit per unit time
- π: Shortages cost per unit per unit time
- S: Selling price per unit and
- $(\tau+\eta Q^{v})$: Demand rate

III. ASSUMPTIONS OF THE MODEL

For developing the model the following assumptions are made

- The demand is known and production quantity dependent demand rate is $(\tau+\eta Q^{v})$
- The rate of production R (t) is time dependent and follows a power pattern.

  i.e., $R(t) = \dfrac{r\, t^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}$,
- where, r is the total production and n is the index parameter.
- Lead time is zero
- Cycle length is known and fixed say T
- Shortages are allowed and fully back logged
- A deteriorated unit is lost, and there is no repair or replacement of the deteriorated unit
- The life time of the commodity is a random and follows an exponential distribution, then the instantaneous rate of deterioration is θ.

IV. PRODUCTION LEVEL INVENTORY MODEL WITH SHORTAGES

In this model the stock level is zero at time t = 0. The stock level increases during the period $(0, t_1)$ due to excess production after fulfilling the demand and deterioration. The production stops at time t_1 when the stock level reaches S. The inventory decreases gradually due to demand and deterioration in the interval (t_1, t_2). At time t_2 the inventory reaches zero and the back orders get accumulated during the period (t_2, t_3). At time t_3 the production again starts and fulfils the backlog after satisfying the demand during (t_3, T), and the inventory level has increased. The schematic diagram representing the instantaneous state of inventory is given in Figure 1.1

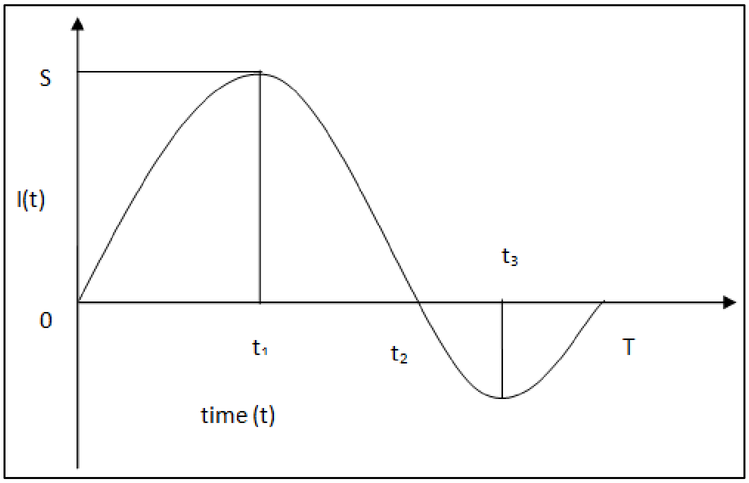


Fig. 1.1: Schematic diagram representing the inventory level.

Let I(t) be the inventory level of the system at time t ($0 \le t \le T$).
The differrential equations governing the instanteneous state of inventory over the cycle of lenth T are

$$\frac{d}{dt}I(t) + \theta I(t) = \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} - (\tau + \eta Q^V), \qquad 0 \le t \le t_1 \tag{1}$$

$$\frac{d}{dt}I(t) + \theta I(t) = -(\tau + \eta Q^v), \qquad t_1 \le t \le t_2 \tag{2}$$

$$\frac{d}{dt}I(t) = -(\tau + \eta Q^v), \qquad t_2 \le t \le t_3 \tag{3}$$

$$\frac{d}{dt}I(t) = \frac{r\,t^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} - (\tau + \eta Q^v), \qquad t_3 \le t \le T \tag{4}$$

with the initial conditions $I(t_1) = S$, $I(t_2) = 0$, $I(T) = 0$
Solving the differential equations (1) to (4) we get the on hand inventory at time t as

$$I(t) = e^{-\theta t}\left[\int_t^{t_1}(\tau + \eta Q^v)e^{\theta u}du - \int_t^{t_1}\frac{r\,u^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}e^{\theta u}du + Se^{\theta t_1}\right], \qquad 0 \le t \le t_1 \tag{5}$$

$$I(t) = e^{\theta(t_1 - t)}\left[S + \frac{(\tau + \eta Q^v)}{\theta}\right] - \frac{(\tau + \eta Q^v)}{\theta}, \qquad t_1 \le t \le t_2 \tag{6}$$

$$I(t) = (\tau + \eta Q^v)(t_2 - t), \qquad t_2 \le t \le t_3 \tag{7}$$

$$I(t) = \frac{r}{T^{\frac{1}{n}}}\left[t^{\frac{1}{n}} - T^{\frac{1}{n}}\right] - (\tau + \eta Q^v)(t - T), \qquad t_3 \le t \le T \tag{8}$$

The stock loss due to deterioration in the interval (0, t) is

$$L(t) = \int_0^t R(t)dt - \int_0^t (\tau + \eta Q^v)dt - I(t), \qquad 0 \le t \le t_2$$

Therefore the stock loss due to deterioration in the cycle of length T is

$$L(T) = \frac{r\,t_1^{\frac{1}{n}}}{T^{\frac{1}{n}}} - (\tau + \eta Q^v)t_2 .$$

Production quantity Q in the cycle of length T is

$$Q = \frac{r}{T^{\frac{1}{n}}}\left[t_1^{\frac{1}{n}} + T^{\frac{1}{n}} - t_3^{\frac{1}{n}}\right]. \tag{9}$$

From the equations (5) and using the initial condition I (0) = 0 we get the value of S as

$$S = e^{-\theta t_1}\left[-\int_0^{t_1}(\tau + \eta Q^v)\,e^{\theta u}du + \int_0^{t_1}\frac{r\,u^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}e^{\theta u}du\right]. \tag{10}$$

From equation (6) and using the condition I (t_2) = 0 we get

$$e^{\theta(t_1 - t_2)}\left(S\theta + (\tau + \eta Q^v)\right) = (\tau + \eta Q^v). \tag{11}$$

Substitute the value of S from equation (10) in equation (11) and on simplification we get t_2 in terms of t_1 as

$$t_2 = \frac{1}{\theta}\log\left\{\left[\frac{r\,\theta}{(\tau + \eta Q^v)T^{\frac{1}{n}}}\left(t_1^{\frac{1}{n}} + \frac{\theta t_1^{\frac{1}{n}+1}}{n+1}\right)\right] + 1\right\} = x(t_1) \quad (say) \tag{12}$$

Taking $t = t_3$ in the equations (7) and (8) and equating these, we get

$$t_3 = \left[T^{\frac{1}{n}} - \frac{(\tau + \eta Q^v)}{r} T^{\frac{1}{n}} \left(T - t_2 \right) \right]^n \quad . \tag{13}$$

Substituting the value of t_2 from the equation (12) in equation (13) we get

t_3 in terms of t_1 ,

$$t_3 = \left(T^{\frac{1}{n}} - y\ (t_1) \right)^n \quad . \tag{14}$$

$$\text{where} \quad y(t_1) = \frac{(\tau + \eta Q^v) T^{\frac{1}{n}}}{r} \left(T - x(t_1) \right)$$ and x (t_1) is as given in equation (12)

Let K (t_1, t_2, t_3) be the total cost per unit time. Since the total cost is sum of the set up cost, cost of the units the inventory holding cost and shortage cost, K (t_1, t_2, t_3) becomes

$$K(t_1, t_2, t_3) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left[\int_0^{t_1} I(t)\ dt + \int_{t_1}^{t_2} I(t)\ dt \right] + \frac{\pi}{T}\left[\int_{t_2}^{t_3} -I(t)\ dt + \int_{t_3}^{T} -I(t)\ dt \right] . \tag{15}$$

Substituting the values of I (t) and Q from equations (5), (6), (7),(8) and (9) in equation(15) we get

$$\begin{aligned} K(t_1,t_2,t_3) = {} & \frac{A}{T} + \frac{C\ r}{T^{\frac{n+1}{n}}}\left[t_1^{\frac{1}{n}} + T^{\frac{1}{n}} - t_3^{\frac{1}{n}} \right] + \frac{h}{T}\left\{ \frac{e^{\theta t_1}}{\theta}\left(S + \frac{(\tau + \eta Q^v)}{\theta} \right)\left(1 - e^{-\theta t_2}\right) + \frac{r}{T^{\frac{1}{n}}}\left[\frac{t_1^{\frac{1}{n}}}{\theta}(e^{-\theta t_1} - 1) \right.\right. \\ & \left.\left. + \frac{n\ t_1^{\frac{1}{n}+1}}{(n+1)}(1 - \frac{1}{n} + \frac{1}{n}e^{-\theta t_1}) + \frac{n^2 \theta t_1^{\frac{1}{n}+2}}{(n+1)(1+2n)} - \frac{n\theta^2 t_1^{\frac{1}{n}+3}}{(n+1)(1+3n)} \right] - \frac{(\tau + \eta Q^v)}{\theta} t_2 \right\} \\ & + \frac{\pi}{T}\left\{ \frac{(\tau + \eta Q^v)}{2}\left[2t_3\left(T - t_2\right) + \left(t_2^{\ 2} - T^2\right) \right] + \frac{r}{T^{\frac{1}{n}}}\left[\frac{T^{\frac{1}{n}+1}}{(n+1)} - T^{\frac{1}{n}} t_3 + \frac{n t_3^{\frac{1}{n}+1}}{(n+1)} \right] \right\} . \end{aligned} \tag{16}$$

Substituting the values of S, t_2 and t_3 from equations(10), (12) and (14) in equations (16) , K (t_1, t_2, t_3) becomes K (t_1)

$$\begin{aligned} K(t_1) = {} & \frac{A}{T} + \frac{C\ r}{T^{\frac{n+1}{n}}}\left[t_1^{\frac{1}{n}} + [T^{\frac{1}{n}} - y(t_1)]^{\frac{1}{n}} \right] + \frac{h}{T}\left\{ \frac{r}{T^{\frac{1}{n}}}\left(\frac{t_1^{\frac{1}{n}}}{\theta} + \frac{t_1^{\frac{1}{n}+1}}{(n+1)} + \frac{(\tau + \eta Q^v)}{\theta^2} \right)\left(1 - e^{-\theta x(t_1)}\right) + \frac{r}{T^{\frac{1}{n}}}\left[\frac{t_1^{\frac{1}{n}}}{\theta}(e^{-\theta t_1} - 1) \right.\right. \\ & \left.\left. + \frac{n\ t_1^{\frac{1}{n}+1}}{(n+1)}(1 - \frac{1}{n} + \frac{1}{n}e^{-\theta t_1}) + \frac{n^2\theta\ t_1^{\frac{1}{n}+2}}{(n+1)\ (1+2n)} - \frac{n\ \theta^2\ t_1^{\frac{1}{n}+3}}{(n+1)\ (1+3n)} \right] - \frac{(\tau + \eta Q^v) x(t_1)}{\theta} \right\} \\ & + \frac{\pi}{T}\left\{ \frac{(\tau + \eta Q^v)}{2}\left[2[T^{\frac{1}{n}} - y(t_1)]^n \left(T - x(t_1)\right) + \left((x(t_1))^2 - T^2\right) \right] \right. \\ & \left. + \frac{r}{T^{\frac{1}{n}}}\left[\frac{T^{\frac{1}{n}+1}}{(n+1)} - T^{\frac{1}{n}}[T^{\frac{1}{n}} - y(t_1)]^n + \frac{n}{(n+1)}[T^{\frac{1}{n}} - y(t_1)] \right]^{\frac{1}{n}+1} \right\} . \end{aligned} \tag{17}$$

OPTIMAL POLICIES OF THE MODEL In this section, the optimal policies of the inventory system are derived. To find the optimal values of t1, we minimize the total cost per unit time with respect to t1.The conditions for optimal value of t1 are, $\frac{dK(t_1)}{dt_1} = 0 \quad \text{and} \quad \frac{d^2K(t_1)}{dt_1^{\ 2}} > 0$ Differentiating K (t1) given in equation (17)with respect to t1 and equating it to zero we get

$$\frac{C\,r}{T^{\frac{1}{n}}}\left(\frac{t_1^{\frac{1}{n}-1}}{n}-(\tau+\eta Q^{v})\right)\frac{T^{\frac{1}{n}}}{r}z_1(t_1)+h\left\{\frac{r}{n\,\theta T^{\frac{1}{n}}}\left(e^{-\theta t_1}-e^{-\theta x(t_1)}\right)t_1^{\frac{1}{n}-1}+\frac{r}{T^{\frac{1}{n}}}\left(1+e^{-\theta x(t_1)}\left(z_1(t_1)-\frac{1}{n}\right)\right.\right.$$

$$\left.-\left(1-\frac{1}{n}\right)e^{-\theta t_1}\right)t_1^{\frac{1}{n}}+\frac{r\,n\theta}{(n+1)T^{\frac{1}{n}}}\left[\frac{1}{n}e^{-\theta x(t_1)}z_1(t_1)-\left(1+\frac{1}{n}e^{-\theta t_1}\right)\right]t_1^{\frac{1}{n}+1}-\frac{r\,\theta^{2}}{(n+1)T^{\frac{1}{n}}}t_1^{\frac{1}{n}+2}+$$ where,

$$\left.+\frac{(\tau+\eta Q^{v})}{\theta}z_1(t_1)\left(e^{-\theta x(t_1)}-1\right)\right\}+\pi\,(\tau+\eta Q^{v})\,z_1(t_1)\left\{x(t_1)+\left(T^{\frac{1}{n}}-y(t_1)\right)^{n-1}\right.$$

$$\left.\left[(\tau+\eta Q^{v})\frac{T^{\frac{1}{n}}}{r}n\left(T-x(t_1)\right)-\left(T^{\frac{1}{n}}-y\right)-y(t_1)n\right]\right\}=0\ . \qquad (18)$$

$x(t_1)$ and $y(t_1)$ are as given in equation(12) and(14) and $z_1(t_1)=\dfrac{\partial x(t_1)}{\partial t_1}$

Solving the equation (18) for t_1 using the numerical methods we get the optimal value of t_1 as t_1^*.The optimum time t_3^* of t_3 at which the production should be restarted is obtained by substituting the optimal value of t_1^* in equation (14), therefore the optimal time at which the production is to be started is

$$t_3^{*}=\left\{T^{\frac{1}{n}}-(\tau+\eta Q^{v})\frac{T^{\frac{1}{n}}}{r}\left[T-\frac{1}{\theta}\log\frac{r\,\theta}{T^{\frac{1}{n}}(\tau+\eta Q^{v})}\left(t_1^{*\frac{1}{n}}+\frac{\theta\,t_1^{*\left(\frac{1}{n}+1\right)}}{(n+1)}\right)+1\right]\right\}^{n}\ . \qquad (19)$$

The optimum production quantity Q^* of Q in the cycle of length T is obtained by substituting the optimal values of $t_1^*$and t_3^* in equation (9) Therefore the optimal production quantity is

$$Q^{*}=\frac{r}{T^{\frac{1}{n}}}\left\{t_1^{*\frac{1}{n}}+\frac{(\tau+\eta Q^{v})T^{\frac{1}{n}}}{r}\left[T-\frac{1}{\theta}\ln\frac{r\,\theta}{(\tau+\eta Q^{v})T^{\frac{1}{n}}}\left(t_1^{*\frac{1}{n}}+\frac{\theta\,t_1^{*\left(\frac{1}{n}+1\right)}}{(n+1)}\right)+1\right]\right\}\ . \qquad (20)$$

V. NUMERICAL ILLUSTRATION

In this section, let us consider the case of deriving optimal production quantity, production down time and production uptime of an industry. Here, it is assumed that the product is of deteriorating nature, shortages are allowed and fully backlogged. For demonstrating the solution procedure of the model, the deteriorating parameter θ is considered to vary as 0.1, 0.11, 0.12, 0.13 and 0.14, the values of other parameters and costs are associated with the model. Then substitute the values in the optimal production quantity Q^*. Production downtime, production uptime and optimal cost of production are computed and presented in Table1. From Table 1, it is observed that the deterioration parameter and production parameters have a tremendous influence on the optimal values of the model. As the deteriorating parameter θ various from 0.10 to 0.14, the optimal production quantity Q^* decreases from 208.324 to 201.087, the optimal value of the production down time decreases from 2.160 to 1.565, the optimal value of production uptime decreases from 4.160 to 3.913 and the total cost of production per unit time K increases from 304.127 to 305.141 units, this increase is nominal.

Table – 1
OPTIMAL VALUES OF t_1^*, t_3^*, Q^* AND K

C	*h*	*π*	*r*	*n*	*θ*	*τ*	*η*	*v*	*A*	*T*	*t₁**	*t₃**	*Q**	*K*
10	0.1	0.3	250	1.0	0.1	15.0	0.1	0.1	1500	12	2.160	4.160	208.324	304.127
11	0.1	0.3	250	1.0	0.1	15.0	0.1	0.1	1500	12	1.984	4.090	206.118	319.718
12	0.1	0.3	250	1.0	0.1	15.0	0.1	0.1	1500	12	1.836	4.031	204.272	335.259
13	0.1	0.3	250	1.0	0.1	15.0	0.1	0.1	1500	12	1.709	3.980	202.694	350.754
14	0.1	0.3	250	1.0	0.1	15.0	0.1	0.1	1500	12	1.600	3.935	201.344	366.226
10	0.2	0.3	250	1.0	0.1	15.0	0.1	0.1	1500	12	2.030	4.109	206.693	302.915
10	0.3	0.3	250	1.0	0.1	15.0	0.1	0.1	1500	12	1.911	4.061	205.206	301.818
10	0.4	0.3	250	1.0	0.1	15.0	0.1	0.1	1500	12	1.801	4.017	203.836	300.813
10	0.5	0.3	250	1.0	0.1	15.0	0.1	0.1	1500	12	1.701	3.977	202.595	299.907
10	0.1	0.4	250	1.0	0.1	15.0	0.1	0.1	1500	12	2.746	4.390	215.748	311.057
10	0.1	0.5	250	1.0	0.1	15.0	0.1	0.1	1500	12	3.295	4.600	222.815	317.235

10	0.1	0.6	250	1.0	0.1	15.0	0.1	0.1	1500	12	3.816	4.794	229.618	322.771
10	0.1	0.7	250	1.0	0.1	15.0	0.1	0.1	1500	12	4.314	4.976	236.207	327.735
10	0.1	0.3	275	1.0	0.1	15.0	0.1	0.1	1500	12	1.997	4.880	208.934	306.023
10	0.1	0.3	300	1.0	0.1	15.0	0.1	0.1	1500	12	1.847	5.475	209.311	307.508
10	0.1	0.3	325	1.0	0.1	15.0	0.1	0.1	1500	12	1.713	5.976	209.553	308.713
10	0.1	0.3	350	1.0	0.1	15.0	0.1	0.1	1500	12	1.594	6.404	209.707	309.712
10	0.1	0.3	250	1.1	0.1	15.0	0.1	0.1	1500	12	1.069	3.405	198.200	295.716
10	0.1	0.3	250	1.2	0.1	15.0	0.1	0.1	1500	12	0.501	2.848	192.316	290.236
10	0.1	0.3	250	1.3	0.1	15.0	0.1	0.1	1500	12	0.244	2.433	189.251	286.815
10	0.1	0.3	250	1.4	0.1	15.0	0.1	0.1	1500	12	0.124	2.101	187.527	284.418
10	0.1	0.3	250	1.0	0.11	15.0	0.1	0.1	1500	12	1.972	4.082	206.038	304.167
10	0.1	0.3	250	1.0	0.12	15.0	0.1	0.1	1500	12	1.815	4.017	204.129	304.407
10	0.1	0.3	250	1.0	0.13	15.0	0.1	0.1	1500	12	1.681	3.961	202.499	304.749
10	0.1	0.3	250	1.0	0.14	15.0	0.1	0.1	1500	12	1.565	3.913	201.087	305.141
10	0.1	0.3	250	1.0	0.1	15.5	0.1	0.1	1500	12	2.203	3.843	214.790	309.195
10	0.1	0.3	250	1.0	0.1	16.0	0.1	0.1	1500	12	2.238	3.623	221.155	314.151
10	0.1	0.3	250	1.0	0.1	16.5	0.1	0.1	1500	12	2.265	3.349	227.420	318.994
10	0.1	0.3	250	1.0	0.1	17.0	0.1	0.1	1500	12	2.281	3.070	233.550	323.696
10	0.1	0.3	250	1.0	0.1	15.0	0.2	0.1	1500	12	2.175	4.069	210.536	305.865
10	0.1	0.3	250	1.0	0.1	15.0	0.3	0.1	1500	12	2.190	3.978	212.752	307.601
10	0.1	0.3	250	1.0	0.1	15.0	0.4	0.1	1500	12	2.204	3.886	214.960	309.328
10	0.1	0.3	250	1.0	0.1	15.0	0.5	0.1	1500	12	2.217	3.793	217.159	311.043
10	0.1	0.3	250	1.0	0.1	15.0	0.1	0.2	1500	12	2.171	4.096	209.892	305.359
10	0.1	0.3	250	1.0	0.1	15.0	0.1	0.3	1500	12	2.189	3.985	212.579	307.466
10	0.1	0.3	250	1.0	0.1	15.0	0.1	0.4	1500	12	2.217	3.791	217.208	311.082
10	0.1	0.3	250	1.0	0.1	15.0	0.1	0.5	1500	12	2.257	3.443	225.293	317.354
10	0.1	0.3	250	1.0	0.1	15.0	0.1	0.1	2000	12	2.160	4.160	208.324	345.794
10	0.1	0.3	250	1.0	0.1	15.0	0.1	0.1	2500	12	2.160	4.160	208.324	387.460
10	0.1	0.3	250	1.0	0.1	15.0	0.1	0.1	3000	12	2.160	4.160	208.324	429.127
10	0.1	0.3	250	1.0	0.1	15.0	0.1	0.1	3500	12	2.160	4.160	208.324	470.794

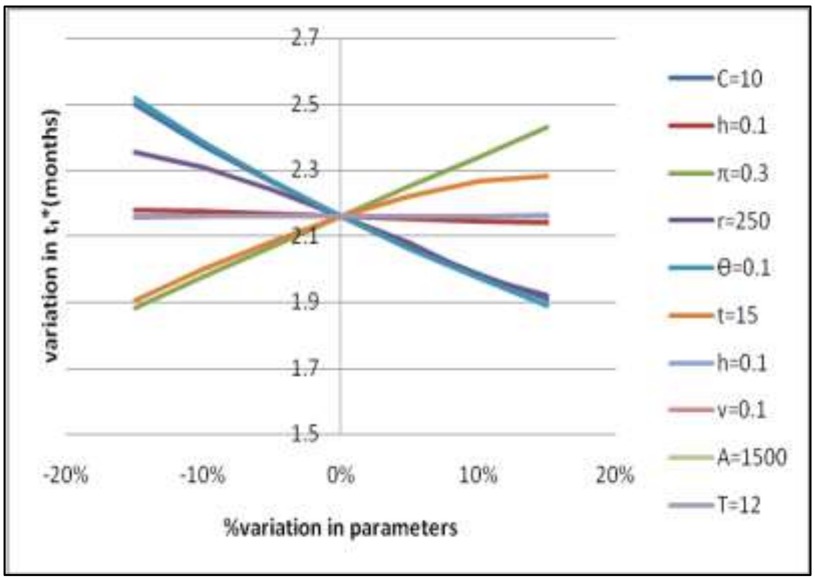
VI. SENSITIVITY ANALYSIS OF THE MODEL

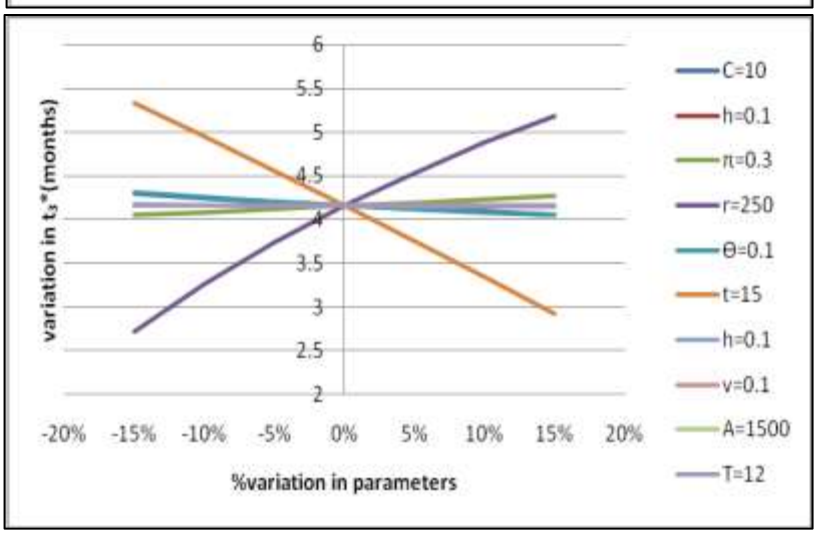
Sensitivity analysis is carried out to explore the effect of changes in parameters and costs on the optimal policies by varying each parameter (-15%, -10%, -5%, 5%, 10%, 15%) at a time and all parameters together for the model under study. The results obtained are presented in Table .2.The relationship between the parameters, cost on the optimal values of the production schedule are shown in Figure .2. It is observed that the variation in the deterioration parameter θ and the demand parameters τ and η have significant influence on optimal production quantity Q^*. As θ increases, the production quantity Q^* is decreasing and the production down time and production uptime are also decreasing when other parameters remain fixed. The production rate parameters have significant influence on the optimal values of the production quantity Q^* and production downtime t_1^* and production uptime t_3^*. As r increases the value of t_1^*, is decreasing, t_3^*, Q^* are increasing. The decrease in t_1^* is marginal and the increase in t_3^* is rapid.

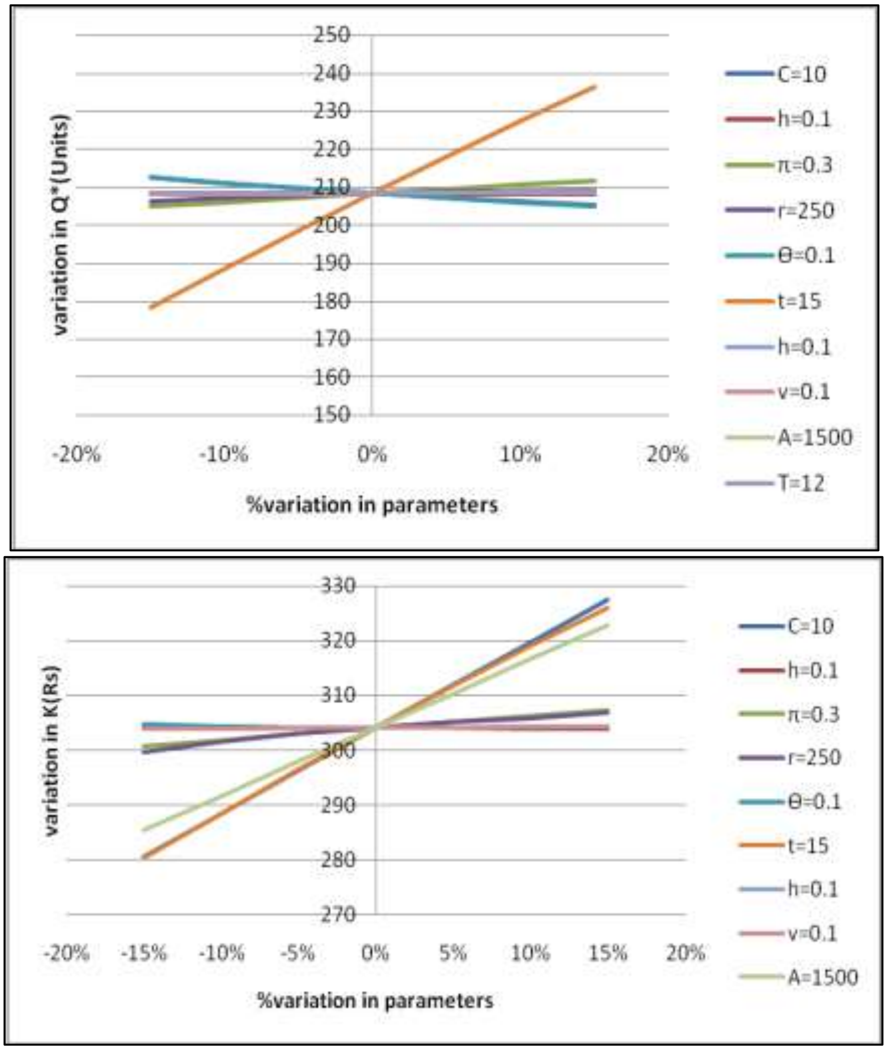
Table – 2
Sensitivity Analysis of the Model With Respect to Parameters and Costs

Variation in Parameters	*Optimal Policies*	*Percentage Change in Parameters*						
		-15	-10	-5	0	5	10	15
C	t_1^*	2.497	2.373	2.261	2.160	2.068	1.984	1.907
	t_3^*	4.293	4.244	4.200	4.160	4.124	4.090	4.059
	Q*	212.579	211.008	209.595	208.324	207.17	206.118	205.157
	K	280.615	288.474	296.308	304.127	311.929	319.718	327.494
h	t_1^*	2.180	2.173	2.166	2.160	2.153	2.146	2.139
	t_3^*	4.168	4.166	4.163	4.160	4.158	4.155	4.152
	Q*	208.575	208.487	208.399	208.324	208.236	208.148	208.06
	K	304.315	304.249	304.183	304.127	304.061	303.996	303.931
π	t_1^*	1.881	1.975	2.068	2.160	2.250	2.340	2.429
	t_3^*	4.049	4.087	4.124	4.160	4.196	4.232	4.266
	Q*	204.832	206.006	207.17	208.324	209.456	210.591	211.717
	K	300.724	301.878	303.012	304.127	305.214	306.29	307.347
n	t_1^*	4.329	3.603	2.865	2.160	1.546	1.069	0.730
	t_3^*	5.480	5.038	4.593	4.160	3.759	3.405	3.104
	Q*	225.908	220.36	214.373	208.324	202.758	198.2	194.774
	K	317.77	315.554	308.917	304.127	299.59	295.716	292.629

r	t_1*	2.355	2.310	2.240	2.160	2.078	1.977	1.920
	t_3*	2.717	3.263	3.738	4.160	4.538	4.880	5.191
	Q*	206.095	207.142	207.841	208.324	208.676	208.655	209.143
	K	299.639	301.478	302.932	304.127	305.144	305.827	306.806
θ	t_1*	2.518	2.386	2.267	2.160	2.062	1.972	1.890
	t_3*	4.310	4.255	4.205	4.160	4.120	4.082	4.048
	Q*	212.674	211.071	209.624	208.324	207.133	206.038	205.041
	K	304.773	304.418	304.213	304.127	304.118	304.167	304.268
τ	t_1*	1.905	1.998	2.084	2.160	2.221	2.265	2.283
	t_3*	5.340	4.95	4.557	4.160	3.758	3.349	2.929
	Q*	178.431	188.502	198.478	208.324	217.978	227.42	236.542
	K	280.296	288.396	296.349	304.127	311.682	318.994	325.976
η	t_1*	2.157	2.158	2.159	2.160	2.160	2.161	2.162
	t_3*	4.174	4.169	4.165	4.160	4.156	4.151	4.147
	Q*	207.983	208.097	208.21	208.324	208.425	208.539	208.652
	K	303.859	303.949	304.038	304.127	304.207	304.296	304.385
v	t_1*	2.158	2.159	2.159	2.160	2.160	2.160	2.161
	t_3*	4.167	4.165	4.163	4.160	4.158	4.155	4.153
	Q*	208.143	208.206	208.258	208.324	208.379	208.435	208.505
	K	303.986	304.035	304.076	304.127	304.17	304.215	304.27
A	t_1*	2.160	2.160	2.160	2.160	2.160	2.160	2.160
	t_3*	4.160	4.160	4.160	4.160	4.160	4.160	4.160
	Q*	208.324	208.324	208.324	208.324	208.324	208.324	208.324
	K	285.377	291.627	297.877	304.127	310.377	316.627	322.877







With Shortages

Fig.1.2 The graphical representation of sensitivity analysis of production quantity dependent demand – with shortages

VII. CONCLUSION

Production inventory models play a dominant role in production scheduling and resource allocation. In this paper, to develop inventory models for deteriorating items with time dependent production having production quantity dependent demand having power pattern and follows an exponential distribution. The models were illustrated with numerical examples and sensitivity analysis of the models with respect to cost and parameters was also carried out. Different types of production having optimal order quantities, optimal up time, optimal down time and optimal system profit were obtained for different choices of costs and parameters. It can be concluded from the numerical examples and sensitivity analysis that the production quantity dependent nature of production rate is having significant influence on the optimal production quantity and production up-time, production down time and the demand parameters tremendously influence the optimal values of the total production rate. The models are developed by assuming that the money value remains constant throughout the period of time. It is also possible to develop the EPQ models discussed in this paper under inflation (time value of money). The models developed for single item can also be extended to include multiple commodities.

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