Special Class Extended Mean Corial Graphs

Dr. A. NellaiMurugan
Department of Mathematics
V. O. Chidambaram College, Tuticorin 628008, India

S. Soniya
Department of Mathematics
V. O. Chidambaram College, Tuticorin 628008, India

Abstract

Let $G = (V,E)$ be a graphs with $p$ vertices and $q$ edges. An Extended Mean Cordial Labeling of a Graph $G$ with vertex set $V$ is a bijection from $V$ to $\{-1,0,1\}$ such that each edge $uv$ is assigned the label $(|f(u)+f(v)|)/2$ where $|x|$ is the least integer greater than or equal to $x$ with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by almost 1. The graph that admits an Extended Mean Cordial Labeling is called Extended Mean Cordial Graph. In this paper, we proved that special graphs (umbrella), $Q(n)$, $(P_n; C_3)$ are Extended Mean Cordial Graphs.

Keywords: Extended Mean Cordial Graph, Extended Mean Cordial Labeling

2000 mathematics Subject classification 05C78

I. INTRODUCTION

A graph $G$ is finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of $G$ which is called edges. Each pair $e = \{u,v\}$ of vertices in $E$ is called edges or a line of $G$. In this paper, we proved that special graphs (umbrella), $Q(n)$, $(P_n; C_3)$ are Extended Mean Cordial Graphs. For graph theory terminology we follow[2].

II. PRELIMINARIES

Let $G = (V,E)$ be a graphs with $p$ vertices and $q$ edges. A Extended Mean Cordial Labeling of a Graph $G$ with vertex set $V$ is a bijection from $V$ to $\{-1,0,1\}$ such that each edge $uv$ is assigned the label $(|f(u)+f(v)|)/2$ where $|x|$ is the least integer greater than or equal to $x$ with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by almost 1. The graph that admits an Extended Mean Cordial Labeling is called Extended Mean Cordial Graph. In this paper, we proved that special graphs (umbrella), $Q(n)$, $(P_n; C_3)$ are Extended Mean Cordial Graphs.

A. Definition 2.3.1

Umbrella is a graph obtained from a Fan by joining a path of length $m$, $P_m$ to a middle vertex of a path $P_n$ is Fan $F_n$. It is denoted by $U(m,n)$.

B. Definition 2.3.2

A Quadrilateral snake is a graph obtained from the path $P_n$ by replacing every edge of the path with $C_4$. It is denoted by $Q_n$.

C. Definition 2.3.3

If is graph obtained from path by joining each vertex og a path through an edge to any one of the vertex cycle $C_4$.

III. MAIN RESULT

A. Theorem: 3.1

Graph (umbrella) is a extended mean cordial graph

1) Proof:

Let $V(\ ) = \{w_1,w_2,w_3,v_i:1\leq i\leq n\}$

Let $E(\ ) = \{(w_1v_1):1\leq i\leq n\}U\{(w_1w_2)U(w_2w_3)\}$

Define $f: V(\)\to\{-1,0,1\}$ by

The vertex labeling are

$$f(w_1) = 1, 1\leq i\leq n$$

$$f(v_i) = \begin{cases} -1 & \text{if } i \equiv 1 \mod 2, 1 \leq i \leq n \\ 0 & \text{if } i \equiv 0 \mod 2, 1 \leq i \leq n \\ \end{cases}$$

$$f(w_2) = 1, 1\leq i\leq n$$

$$f(w_2) = 0, 1\leq i\leq n$$
The induced edge labeling is
\[ f^*(w_1v_i) = 0, \quad 1 \leq i \leq n \]
\[ f^*(w_1w_2) = 1, \quad 1 \leq i \leq n \]
\[ f^*(w_2w_3) = 0, \quad 1 \leq i \leq n \]

Here, When \( n = 2m \)
\[ ef(0) = 2m+1, \quad ef(1) = 2m \]

When \( n = 2m+1 \)
\[ ef(0) = 2m+2, \quad ef(1) = 2m+1 \]

Hence, umbrella satisfies the condition \(|ef(0)-ef(1)| \leq 1\)
Therefore, umbrella is extended mean cordial graph
For example, umbrella is extended mean cordial graphs as shown in the figure 1.

![Fig. 1: (umbrella)](image)

**B. Theorem: 3.3**

Graph \( Q(n) \) is a extended mean cordial graph

1) Proof:
Let \( v(Q(n)) = \{(v_i): 1 \leq i \leq n,(u_{i1},u_{i2}): 1 \leq i \leq n-1\} \)

Let \( E(Q(n)) = \{(v_i,v_{i+1})U(v_i,u_{i1})U(v_{i+1},u_{i2}): 1 \leq i \leq n-1\}U[(u_{i1},u_{i2}): 1 \leq i \leq n-1] \)

Define \( f: v(Q(n)) \rightarrow \{-1,0,1\} \) by

The vertex labeling are
\[ f(u_{i1}) = -1, \quad 1 \leq i \leq n-1 \]
\[ f(u_{i2}) = 0, \quad 1 \leq i \leq n-1 \]
\[ f(v_i) = 1, \quad 1 \leq i \leq n \]

The induced edge labeling are
\[ f^*(u_{i1}u_{i2}) = 1, \quad 1 \leq i \leq n-1 \]
\[ f^*(v_i,v_{i+1}) = 1, \quad 1 \leq i \leq n-1 \]
\[ f^*(v_i,u_{i1}) = 0, \quad 1 \leq i \leq n-1 \]
\[ f^*(v_{i+1},u_{i2}) = 0, \quad 1 \leq i \leq n-1 \]

Here, When \( n = 2m \)
\[ ef(0) = ef(1) = 4m-2 \]

When \( n=2m+1 \)
\[ ef(0) = ef(1) = 4m \]

Hence, \( Q(n) \) is satisfies the condition \(|ef(0)-ef(1)| \leq 1\)
Therefore, \( Q(n) \) is a extended mean cordial graph
For example, \( Q(n) \) is a extended mean cordial graph as shown in figure 2.

![Fig. 2: Q(n)](image)
C. Theorem: 3.5

Graph \( (P_n; C_3) \) is a extended mean cordial graph
Proof:
Let \( v(P_n; C_3) = \{u_i,v_i,w_i;1\leq i\leq n\} \)
Let \( E(P_n; C_3) = {\{(u_i,u_{i+1});1\leq i\leq n-1\}|U(u_i,v_i)U(v_i,v_{i+1})U(v_i,w_i)U(w_i,w_{i+1});1\leq i\leq n|\} \)

Define \( f: v(P_n; C_3) \rightarrow \{-1,0,1\} \) by

The vertex labeling are
\[
\begin{align*}
f(u_i) &= 1, 1 \leq i \leq n \\
f(w_i) &= 1, 1 \leq i \leq n \\
f(v_i) &= -1, 1 \leq i \leq n \\
f(v_1) &= 1, 1 \leq i \leq n \\
f(v_2) &= 0, 1 \leq i \leq n
\end{align*}
\]

The induced edge labeling are
\[
\begin{align*}
f^*(u_iu_{i+1}) &= 1, 1 \leq i \leq n-1 \\
f^*(u_iu_i) &= 0, 1 \leq i \leq n \\
f^*(v_iu_i) &= 0, 1 \leq i \leq n \\
f^*(v_iw_i) &= 0, 1 \leq i \leq n \\
f^*(v_1w_1) &= 1, 1 \leq i \leq n \\
f^*(v_2w_2) &= 0, 1 \leq i \leq n
\end{align*}
\]

Here, When \( n = m \)

\( ef(0) = 3m \), \( ef(1) = 3m-1 \)

Hence, \( (P_n; C_3) \) is satisfies the condition \( |ef(0)-ef(1)| \leq 1 \)
Therefore, \( (P_n; C_3) \) is a extended mean cordial graph
For example, \( (P_n; C_3) \) is a extended mean cordial graph as shown in figure 3.

![Graph](image)

Fig. 3: \( 6(P_n; C_3) \)

REFERENCES


