On the Diophantine Equation $128^x + 961^y = Z^2$

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Abstract

By applying Catalan conjectures given Diophantine equation $128^x + 961^y = z^2$ gives a unique non-negative integer solution, that is (1, 1, 33). And the given Diophantine equation can be reduced in the form of $2^x+31^y=z^2$, where x, y and z are non-negative integer. Which gives the solution (7, 2, 18).

Keywords: Catalan Conjectures, Diophantine Equation

I. Introduction

In 2007, Acu [1] proved that (3, 0, 3) and (2, 1, 3) are only two solutions in non-negative integers of the Diophantine equation $2^x+5^y=z^2$. In 2012, Sroysang [2] proved that the Diophantine equation $32^x + 49^y = z^2$ has non-negative integer (1, 1, 9) is a unique solution. In this paper we will show that the Diophantine equation $128^x + 961^y = z^2$ has non-negative integer (1, 1, 33) is a unique solution.

II. Preliminaries

In 1844, Catalan [3] conjectures that the Diophantine equation $a^x-b^y=1$ has a unique integer solution with min{a, b, x, y}>1. The solution (a, b, x, y) is (3, 2, 2, 3). This conjecture was proven by Mihailescu [4] in 2004

A. Preposition 2.1 ([5])

(3, 2, 2, 3) is a unique solution (a, b, x, y) of the Diophantine equation $a^x+b^y=1$, where a, b, x and y are integers with min{a, b, x, y}>1

B. Preposition 2.2 ([6])

(3, 0, 3) is a solution (x, y, z) of the Diophantine equation $2^x+31^y=z^2$, where x, y and z are non-negative integer.

III. Results

In this section, we prove that the Diophantine equation $128^x + 961^y = z^2$ has a unique non-negative integer solution. The solution (x, y, z) is (1, 1, 33). This result implies that (7, 2, 33) is solution (x, y, z) of the Diophantine equation $2^x+31^y=z^2$, where x, y and z are non-negative integer.

A. Theorem 3.1

(1, 1, 33) is a unique solution (x, y, z) of the Diophantine equation $128^x + 961^y = z^2$, where x, y and z are non-negative integers

1) Proof

We will divide the number x into two cases.

- Case (i) x=0. We focus on the equation $1+961^y=z^2$, then $(z-1)(z+1)=31^{2y}$. Thus $z-1=31^u$, where u is non-negative integer. Then $z+1=31^{2y-u}$. It follows that $31^{2y-u}-31^u=2$. Then$31^u(31^{2y-2u}-1)=2$. This implies that u=0. Then $31^{2y}-1=2$ thus $31^{2y}=3$. This is impossible.
- Case (ii) $x\geq1$, in this case z is odd, we consider the equation $128^x + 961^y = z^2$ as the equation $2^{7x}+31^{2y}=z^2$. Then $(z-31^y)(z+31^y)=2^{7x}$. Then $(z-31^y)=2^w$, where w is non-negative integer. Note that 31^y is odd. We have $w\neq0$. Moreover, $z+31^y=2^{7x-w}$. it follows that $2^{7x-w}-2^w=2(31^y)$. Then $2^w(2^{7x-2w}-1)=2(31^y)$. Then w=1. It follows that $2^{7x-2}-1=31^y$. If y=0. Then x=0.43. Thus, $y\geq1$. By preposition we obtain that x=1 or y=1. Now we note that x=1 if and only if y=1. Thus x=1 and y=1. Then $z^2=128+961=1089$. Hence z=33.

Therefore, (x,y,z)=(1,1,33)

B. Corollary 3.2

(7, 2, 33) is a solution of (x, y, z) of the Diophantine equation $2^x+31^y=z^2$, where x, y and z are non-negative integers

1) Proof

By theorem, we obtain that $128^1+961^1=33^2$. This implies that $2^7+31^2=33^2$. Therefore (7, 2, 33) is a solution of (x, y, z) of the Diophantine equation $2^x+31^y=z^2$, where x, y and z are non-negative integers

IV. OPEN PROBLEM

By Preposition 2.2 and corollary 3.2, we know that (3, 0, 3) and (7, 2, 33) are two solutions (x, y, z) of the Diophantine equation $2^x+31^y=z^2$, where x, y and z are non-negative integers. However, we may question that "what is the set of all solutions (x, y, z) of the Diophantine equation $2^x+31^y=z^2$, where x, y and z are non-negative integers?"

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