Design and Simulations of Sliding Mode Controller for DC–DC Buck Converter

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Abstract

A DC–DC buck converter has become popular in application to converts a high voltage to a low voltage efficiently. In this paper, a Sliding Mode Control is devised using Gao's reaching law for buck converter. MATLAB simulation is performed and simulation results are presented.
Keywords: Buck Converter, State Space Modelling, Sliding Mode Control

I. Introduction

A step-down converter (voltage buck converter) is a DC–DC buck converter which steps down DC voltage while boosting up current from its input to its output. It is a group of switched mode power supply. This power converters are used in several applications like, appliance control, power supply for private computers, aircraft, DC motor drives, telecommunication equipment's, automotive, active filters, etc is illustrated in [2]. Pulse width modulation support for controlling of chopper. The primary DC-DC power converter circuit popular as buck converter is display in Figure 1. The working of such devices is frequently based on the control of the output voltage of a passive filter. The DC-DC step down converter contain switch network that disparate the DC offset of voltage and a low-pass filter that eliminate the high frequency switching harmonics. The input voltage disparity in a buck converter can be sighted as changes in the process gain.

On the other way, switched mode power converters inherently contain switching appliances which displays an intermittent behaviour, the DC-DC step down converter can be modelled as a bilinear system, which attains a dissimilar linear topology for every state of the control signal. From this belief, the DC-DC converter can be examined as an unreliable form system since its form is periodically changed by the movement of the controlled switches.

II. Literature review

The switched mode power converters are circuits in electronic network. They are generally operating to acquire a stable output voltage from a given supply DC voltage which is less from that supply voltage. To plant the control system of a converter, it is compulsory to model the converter energetic behaviour. The state space modelling during ON and OFF state can be formulated with the account of switching duty cycle [3] Modelling of converter energetic behaviour is pannier by changing and PWM operations with nonlinear time varying nature. Preparing the form of a bilinear system of a DC-DC step down converter was explained in [1] Sliding Mode control techniques used in unreliable structure systems construct this systems sturdy to parameter disparity and external interference [4]- [5]. Sliding mode control has a high degree of design flexibility and it is relatively simple to execute. In several industrial applications it has widely used, e.g. automotive control, furnace control, etc. [6]. The execution of sliding mode control for switched mode power converter is discussed [7]- [8]. Mattavelli et al. presented a sliding-mode controller, which is used in several applications of DC-DC power converter configurations. The circuit difficulty is same as current-mode controllers and it gives sturdiness and speed of response in opposition to line and parameter interference [9]. Sira-Ramirez and Rios-Bolivar [11] appealed a method in sliding mode control design that is extended linearization method that has excellent self-scheduling properties. They also discussed to integrate a sliding mode control strategy with passivity-based controllers into the DC-DC power converter that magnify its sturdiness properties.

III. Motivation

The DC-DC Buck Converter are popular in applications like, appliance control, power supply for private computers, aircraft, DC motor drives, telecommunication equipment's, automotive etc. Lots of control techniques are used for control of DC-DC Buck converters. Easy and economical controller structure preferred in most industrial and high-performance applications. The control techniques that gives good results under any state is always in demand. Sliding Mode Control is a robust control technique. A more robust and optimal controller design can be achieved by using sliding mode controller. Sliding Mode control gives better performance and reduce control efforts. Thus, designing a Sliding mode controller for DC-DC buck converter is the motivations for undertaking the work.

This paper is organized in the following manner: Section I. described introduction. Section II. depicts literature review the Section III. talks about motivation. In section IV. mathematical modelling of DC-DC Buck Converter is performed. In section V. Sliding Mode Control is developed. Section VI. displayed simulation results. Section VII. concludes the paper.

IV. Mathematical Modelling of DC-DC Buck Converter

Figure 1 represents DC-DC step down Converter System. To map the control system of a DC-DC step down converter, it is mandatory to design the converter energetic behaviour. The state space modelling during ON and OFF state can be formulated with the account of switching duty cycle [3]

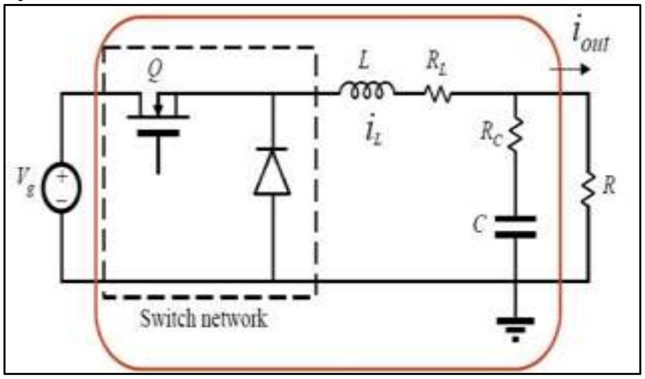

Fig. 1: Buck Converter System

The system parameters are listed below,
V_g : Input voltage (V),
L : Inductance (mH),
C : Capacitance (µF),
R : Resistance(Ω),
i_L : Inductor current (A),
v_c : Voltage across capacitor (V),
v_r : Reference voltage (V),
i_C : Capacitor current,
R_C : Equivalent series resistance of capacitor (Ω),
R_L : Equivalent series resistance of inductor (Ω), F_s : Switching frequency (kHz)

A. Inputs:

- Input voltage (V_g)
- Switching signal u = {0 1}
- Load current (i_{out})

B. Outputs:

- Output voltage (V_o)
- Inductor current (i_L)

In the working principle, the buck converter circuit has two modes of behaviour: 1) The Duty interim during which the converter switch is closed and the voltage input supply is applied across the Load. 2) The Free-Wheeling interim during which the switch is open and the voltage input supply is cut off from the Load. To execute this in model formulation, we multiply the Input Voltage V_g with the pulse width modulation waveform. The result is that the input voltage is included in the Kirchhoff's Voltage Law Equation when the PWM waveform is high ($V_g*1 = V_g$) and excluded when it is low ($V_g*0 = 0$).

The first loop is accountable for computing the inductor current by solving the differential equation we achieved a network structure applying Kirchhoff's Voltage Law to Figure 2

$$V_g u = L\frac{di_L}{dt} + i_L R_L + i_c R_C + v_c \quad (1)$$

$$L\frac{di_L}{dt} = V_g u - i_L R_L - i_c R_C - v_c \quad (2)$$

$$L\frac{di_L}{dt} = V_g u - i_L R_L - (i_c R_C + v_c) \quad (3)$$

$$V_o = i_C R_C + v_c \quad (4)$$

$$L\frac{di_L}{dt} = V_g u - i_L R_L - V_o \quad (5)$$

$$\frac{di_L}{dt} = \frac{1}{L}(V_g u - i_L R_L - V_o) \quad (6)$$

The second loop determine voltage output which is the addition of the voltage across capacitor and the drop across equivalent series resistance of capacitor (Rc). Applying Kirchhoff's Current Law to Figure 1 at the capacitor junction we get:

$$v_c = \frac{1}{C}\int (i_L - i_{out})dt \quad (7)$$

$$\frac{dv_c}{dt} = \frac{1}{C}(i_L - i_{out}) \quad (8)$$

$$i_{out} = \frac{V_o}{R} = \frac{v_c + i_C R_C}{R} \quad (9)$$

$$\frac{dv_c}{dt} = \frac{1}{C}\left(i_L - \frac{v_c + i_C R_C}{R}\right) \quad (10)$$

$$Vo = R\, i_{out} = v_c + R_C\, i_C \quad (11)$$

$$i_C = i_L - i_{out}$$

$$Vo = v_c + R_C\,(i_L - i_{out}) \quad (12)$$

Equations (6) (8) and (12) are system equations. The inductor currents and output voltage are the outputs of the subsystem but we must control only output voltage. Neglecting equivalent series resistance of inductor (R_L) and capacitor (R_C) to design the plant model dynamics. After neglecting equivalent series resistance and simplifying equations (2) and (10) these two equations get modified as below.

$$L\frac{di_L}{dt} = V_g u - v_c \quad (13)$$

$$\frac{di_L}{dt} = \frac{1}{L}(V_g d - v_c) \quad (14)$$

$$\frac{di_L}{dt} = \frac{V_g u}{L} - \frac{v_c}{L} \quad (15)$$

$$\frac{dv_c}{dt} = \frac{1}{C}\left(i_L - \frac{v_c}{R}\right) \quad (16)$$

$$\frac{dv_c}{dt} = \frac{i_L}{C} - \frac{v_c}{RC} \quad (17)$$

Equations (15) and (17) represents plant dynamic equation.

To design Sliding Mode Controller the plant dynamic equations, need to be represented in state space form as

$$\dot{x} = A\,x + B\,V_g\,u \quad (18)$$

where $x \in IR^n$ is the state vector; $A \in IR^{n\times nn}$ and $B \in IR^{n\times m}$ are matrices with constant real entries; u is a scaler control. From equations (15) and (17) the state space model of buck converter is

$$\begin{bmatrix}\dot{i}_L \\ \dot{v}_C\end{bmatrix} = \begin{bmatrix}0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC}\end{bmatrix}\begin{bmatrix}i_L \\ v_C\end{bmatrix} + \begin{bmatrix}\frac{1}{L} \\ 0\end{bmatrix} V_g u$$

and its state space model in controllable canonical form is Differentiating equation (17) w.r.t time, we get $\ddot{v}_c$ as

$$\ddot{v}_c = \frac{1}{C}\dot{i}_L - \frac{1}{RC}\dot{v}_c \quad (19)$$

Putting the value of $\dot{i}_L$ from equation (15), we get

$$\ddot{v}_c = \frac{1}{C}\left[-\frac{1}{L}v_c + \frac{V_g}{L}u\right] - \frac{1}{RC}\dot{v}_c \quad (20)$$

$$\ddot{v}_c = -\frac{1}{LC}v_c + \frac{V_g}{LC}u - \frac{1}{RC}\dot{v}_c \quad (21)$$

$$\begin{bmatrix}\dot{v}_c \\ \ddot{v}_c\end{bmatrix} = \begin{bmatrix}0 & 1 \\ \frac{-1}{LC} & \frac{-1}{RC}\end{bmatrix}\begin{bmatrix}v_c \\ \dot{v}_c\end{bmatrix} + \begin{bmatrix}0 \\ \frac{1}{L}\end{bmatrix} V_g u$$

In general, to achieved positive voltage output only two formations are mandatory which are shown in Figure 2. These formations communicate to dissimilar state space model. The required voltage output is achieved by changing these formations.

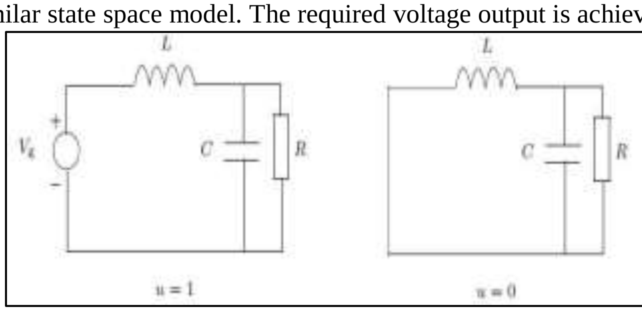

Fig. 2: DC–DC Buck Converter during ON and OFF state

V. CONTROL DEVELOPMENT

Sliding surface for Buck Converter is selected as,

$$S = K_p e + K_i \int e dt + K_d \frac{de}{dt}$$

Where K_p, K_i and K_d are the design parameter to be determined. v_r is the reference voltage across capacitor.

$$\text{error } (e) = v_{ref} - v_c \quad (e) = v_r - v_c \tag{23}$$

$$\dot{e} = -\dot{v}_c \tag{24}$$

$$\ddot{e} = -\ddot{v}_c \tag{25}$$

$$\dot{S} = K_p \dot{e} + K_i e + K_d \ddot{e} \tag{26}$$

Putting value of equations (24), (25) in equation (26) the equation becomes,

$$\dot{S} = -K_p \dot{v}_c + K_i (v_r - v_c) - K_d \ddot{v}_c \tag{27}$$

Putting value of $\dot{v}_c$ from equation (23) in above equation, we get

$$\dot{S} = -K_p \dot{v}_c + K_i (v_r - v_c) - K_d \left[-\frac{1}{LC} v_c + \frac{V_g}{LC} u - \frac{1}{RC} \dot{v}_c \right] \tag{28}$$

Using Gao's Reaching Law

$$\dot{S} = -K \text{ sgn } (S) \tag{29}$$

$$-K sgn(S) = -K_p \dot{v}_c + K_i (v_r - v_c) + \frac{K_d}{LC} v_c - \frac{K_d V_g}{LC} u + \frac{K_d}{RC} \dot{v}_c \tag{30}$$

During sliding motion, S=0, $\dot{S} = 0$

$$-K \text{ sgn } (S) = -K_p \dot{v}_c + K_i (v_r - v_c) - K_d \ddot{v}_c \tag{31}$$

$$u_{SMC} = \frac{v_c}{v_g} + \frac{K_i LC}{K_d V_g}(v_r - v_c) - \frac{L}{K_d V_g}(K_p - \frac{K_d}{RC}) C \dot{v}_c + K sgn(S) \tag{32}$$

$$u_{SMC} = \frac{v_c}{v_g} + \frac{K_i LC}{K_d V_g}(v_r - v_c) - \frac{L}{K_d V_g}\left(K_p C \dot{v}_c - \frac{K_d}{RC} C \dot{v}_c \right) + K sgn(S) \tag{33}$$

Simplifying the above equation, we get

$$u_{SMC} = \frac{v_c}{v_g} + \frac{K_i LC}{K_d V_g}(v_r - v_c) - \frac{L K_p C}{K_d V_g} \dot{v}_c + \frac{L}{V_g R} \dot{v}_c + K sgn(S) \tag{34}$$

This is the integer order sliding mode control equation using PID sliding surface and the simulation result of this controller are shown in Figure 3.

Sliding surface for Buck Converter can be formulated as below,

$$S = K_p e + K_d \frac{de}{dt} \tag{35}$$

With this surface and Gao's reaching law approach, SMC control is designed on the similar basis as above. It arrives at the control law as under

$$\dot{S} = K_p \dot{e} + K_d \ddot{e} \tag{36}$$

Using Gao's Reaching Law

$$\dot{S} = -K \text{ sgn } (S) \tag{37}$$

Putting value of equations (24), (25) and (26) in above equation, the equation becomes,

$$\dot{S} = -K_p \dot{v}_c - K_d \ddot{v}_c \tag{38}$$

Putting value of equation (23) in above equation, we get

$$-K sgn(S) = -K_p \dot{v}_c - K_d \left[-\frac{1}{LC} v_c + \frac{V_g}{LC} u - \frac{1}{RC} \dot{v}_c \right] \tag{39}$$

$$-K sgn(S) = -K_p \dot{v}_c + \frac{K_d}{LC} v_c - \frac{K_d V_g}{LC} u + \frac{K_d}{RC} \dot{v}_c \tag{40}$$

$$u_{SMC} = \frac{v_c}{v_g} - \frac{L}{K_d V_g}(K_p - \frac{K_d}{RC}) C \dot{v}_c + K sgn(S) \tag{41}$$

$$u_{SMC} = \frac{v_c}{v_g} - \frac{L}{K_d V_g}\left(K_p C \dot{v}_c - \frac{K_d}{RC} C \dot{v}_c \right) + K sgn(S) \tag{42}$$

Simplifying the above equation, we get

$$u_{SMC} = \frac{v_c}{v_g} - \frac{LK_pC}{K_dV_g}\dot{v}_c + \frac{L}{V_gR}\dot{v}_c + Ksgn(S) \qquad (43)$$

This is the integer order sliding mode control equation using PD sliding surface and the simulation result of this controller are shown in Figure 4.

A. Proof of Stability:

v_r is the reference voltage and K_p, K_i and K_d are the design parameter to be decided.
In sliding motion, S=0, $\dot{S} = 0$, the closed-loop energetics of v_c becomes

$$0 = -K_p\dot{v}_c + K_i(v_r - v_c) - K_d\ddot{v}_c$$

Dividing both side by K_d

$$0 = -\frac{K_p}{K_d}\dot{v}_c + \frac{K_i}{K_d}(v_r - v_c) - \ddot{v}_c$$

$$0 = -\frac{K_p}{K_d}\dot{v}_c + \frac{K_i}{K_d}v_r - \frac{K_i}{K_d}v_c - \ddot{v}_c$$

$$\ddot{v}_c + \frac{K_p}{K_d}\dot{v}_c + \frac{K_i}{K_d}v_c = \frac{K_i}{K_d}v_r$$

As $v_r = u$

$$\ddot{v}_c + \frac{K_p}{K_d}\dot{v}_c + \frac{K_i}{K_d}v_c = \frac{K_i}{K_d}u$$

The solution of above equation represents asymptotic stability if the following states are satisfied:

$$\frac{K_p}{K_d} > 0, \quad \frac{K_i}{K_d} > 0$$

The presence of the sliding mode need that variation $S\dot{S} < 0$ is satisfied.

VI. SIMULATION RESULTS

Figure 3 represents the symbolic block layout of simulation of DC-DC step down converter.

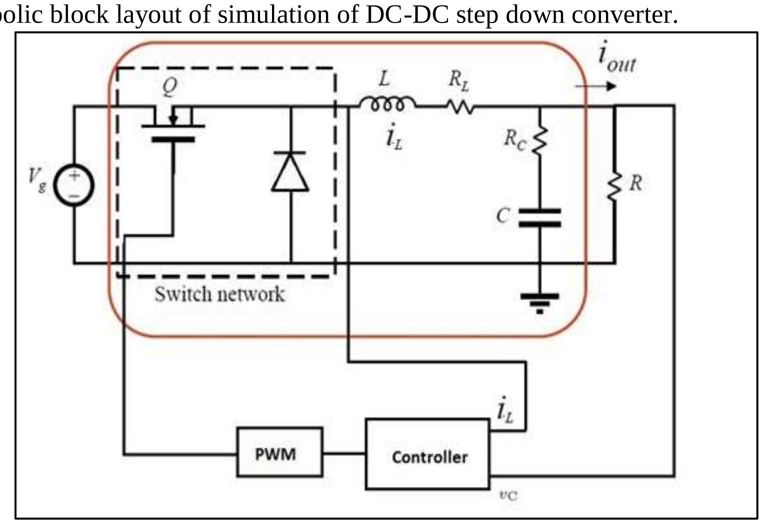

Fig. 3: Schematic Block Diagram of Simulation Model

The values of parameters of buck converter selected for simulations are: R = 7 Ω, L = 4.5mH, C = 47 μF, F_s = 100 kHz, V_g = 20 V, reference voltage v_r = 10 V
MATLAB simulations are performed for above mentioned two cases:

Figure 4 represents the graph of output voltage of buck converter w.r.t time for sliding mode control with Gao's reaching law using PID surface.

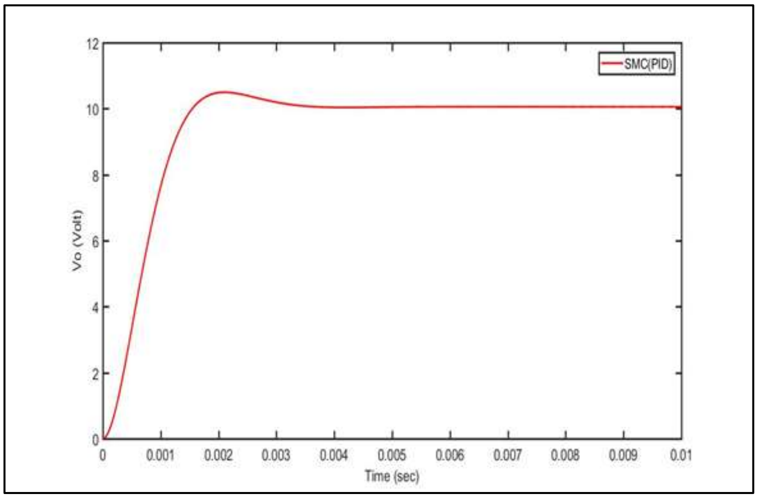

Fig. 4: Output voltage of Buck Converter with SMC using PID surface

Figure 5 depicts the output voltage of buck converter w.r.t time for sliding mode control with Gao's reaching law approach using PD surface.

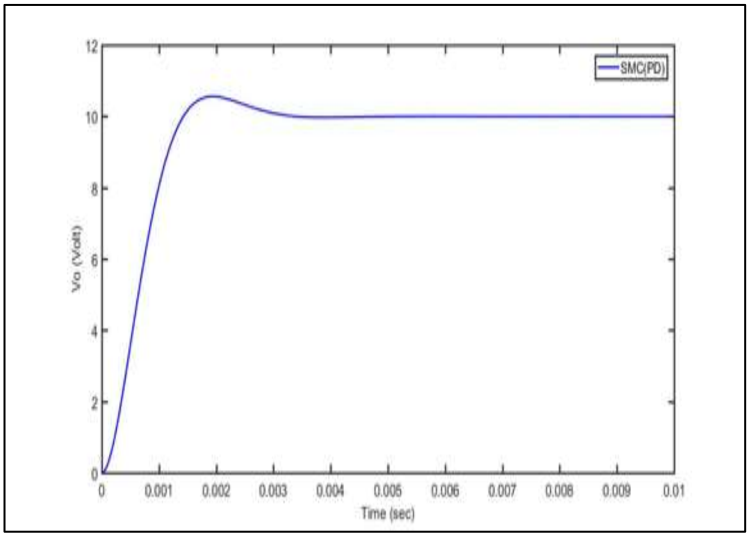

Fig. 5: Output voltage of Buck Converter with SMC using PD surface

To validate the sturdiness of the discussed controller, input voltage is decreased by 15 percent to a value 17 V and simulation is performed. Figure 6 displays the resultant output voltage of buck converter with PD surface. It is seen from Figure 6 that the voltage output follows its reference satisfactorily in spite of the disturbance.

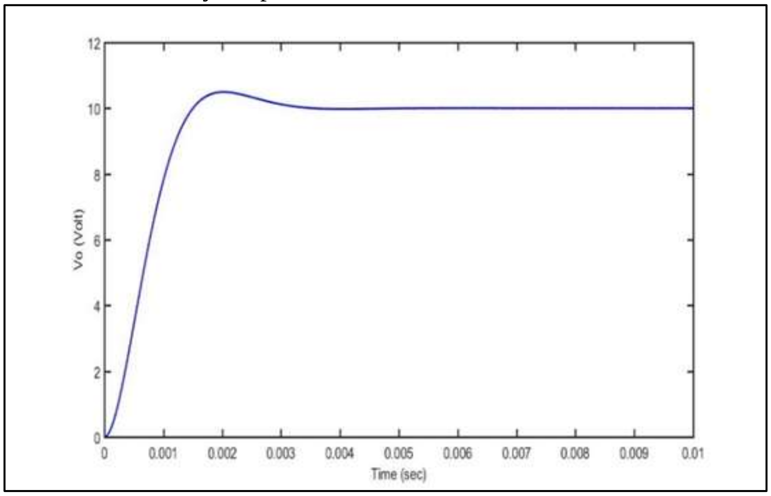

Fig. 6: Output voltage of Buck Converter using SMC with a disturbance in input voltage.

To validate the sturdiness of the discussed controller further, inductor value is decreased by 22 percent to a value 5.5mH and simulation is performed. Figure 7 displays the resultant output voltage of buck converter with PD surface. It is seen from Figure 7 that the output voltage follows its reference satisfactorily in spite of the uncertainty.

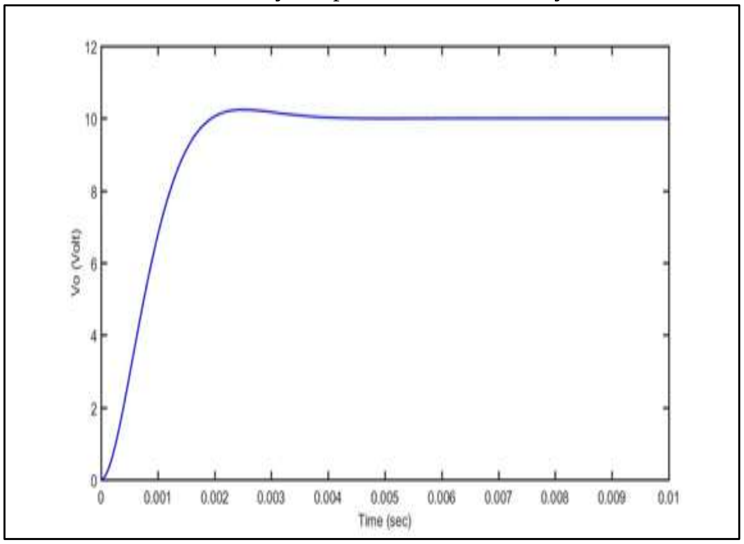

Fig. 7: Output voltage of Buck Converter using SMC with a disturbance in inductor.

To validate the sturdiness of the discussed controller, capacitor value is increased by 25 percent to a value 63 µF and simulation is performed. Figure 8 displays the resultant output voltage of buck converter with PD surface. It is seen from Figure 6 that the voltage output follows its reference satisfactorily in spite of the uncertainty.

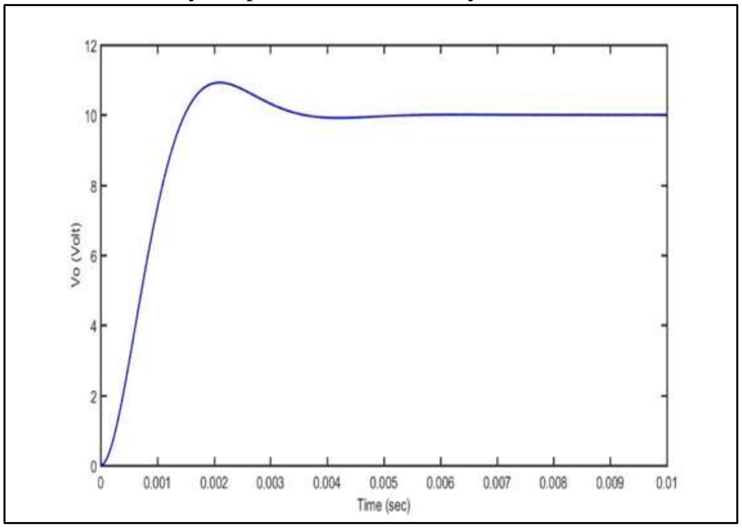

Fig. 8: Output voltage of Buck Converter using SMC with a disturbance in capacitor

VII. CONCLUSIONS

This paper represents, the model of Integer Order Sliding Mode control for DC-DC step down chopper. MATLAB simulations are performed for the proposed Buck Converter system with Sliding Mode Controller (SMC) and results are displayed. Robustness of the proposed controller is verified for disturbances in input voltage as well as for parameter uncertainty.

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