

Implications of Correlations for Sandstone

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Abstract

The objective of this paper is to explore correlations for predicting Uniaxial Compressive Strength and Tangent Modulus – on the basis of compression and shear waves' velocities – for three Himalayan sandstones and implications of using the proposed equations. In this paper, the discussion is done in five stages. First, the representative values of different properties and parameters have been discussed, to have an understanding about the three variants of the sandstone. Second, complete spread of the actual data is shown. Third, correspondence of these values with the equations proposed for sandstone by other researchers has been assessed. Forth, correlations have been explored through regression analysis. Fifth, the implication of using the proposed correlations has been assessed on the same data and the sandstone data of other researchers. This assessment of implication is done by: (1) plotting of actual UCS and estimated UCS from the correlations, (2) calculating percentage error in estimating UCS through these equations, and, (3) assessing variation in the estimated range of values to that of actual. This work reaffirms that the correlations are rock type and location specific that too with certain percentage of errors; in present case percentage of error varies from 30 to 40. However, for preliminary stage of a project, a broad picture of the rock can be obtained using these equations, in the form of range. But, these or any correlations can never be an alternative for the investigation.

Keywords: Sandstone, Laboratory, UCS, Wave Velocity, Modulus, Correlation

I. INTRODUCTION

Uniaxial Compressive Strength (UCS) and Tangent Modulus (E) of intact rock are the basic strength parameters used in the design of foundation and underground structures. Also, UCS is the only laboratory parameter used in Rock Mass Rating System (a geo-mechanical classification system which represents overall comprehensive index of rock mass quality) [Bieniawski, 1989], which is further used for the design and construction of structures in rock. In literature, these parameters have been correlated with compression (V_p) and shear (V_s) waves' velocities in laboratory. Further, these correlations are extrapolated to the geophysical survey to assess rock-mass deformation, damage zones, degree of weathering and rock mass characterisation. Seismic techniques have been used for the assessment of in-situ stresses and rock mass deformation as well as damage zones developed around the underground opening and tunnels (Onodera 1963; Hudson et al 1980; Gladwin 1982). Seismic techniques have also been used for rock mass characterization by assessing the degree of weathering in rock mass (Turk and Dearman 1986; Karpuz and Pasamehmetoglu 1997; Boadu 1997). Also, several researchers [Freyberg (1972); McNally (1987); Hawkins and McConnell 1992; Ulusay et al 1994; Tugrul and Zarif 1999; Kahraman 2001; Yasar and Erdogan 2004; K.B. Chary et al (2006); Sharma and Singh (2007); Kahraman and Yeken (2008); Yagiz (2009)] reported that the V_p has relationship with other mechanical properties of the rock such as UCS, E, Point load strength and others. These correlations have been proposed for different rock variants from different locations, having different ranges of waves' velocity and/or strength and deformability characteristics of rock. Among these correlations, four assessments of UCS based on V_p [Freyberg (1972); McNally (1987); two by K.B. Chary et al (2006)] are given for sandstones.

The objective of this paper is to explore correlations for predicting UCS and E – on the basis of V_p and V_s – for three Himalayan sandstones, namely, SS1, SS2 and SS3. For the purpose, UCS, E and Waves' velocities of all the three sandstones have been assessed. UCS and E have been assessed in saturated state (however, E of SS2 could not be assessed). Although, V_p and V_s have been assessed in both dry and saturated states, but V_p and V_s of saturated samples are taken into consideration because UCS and E are in saturated state.

It may please be noted that the abbreviations of properties and parameters (UCS, E, V_p and V_s) used in this paper refer them in saturated state, unless stated otherwise. Also, at places these abbreviations are suffixed with 'a' or 'e'; 'a' stands for actual value based on investigation and 'e' stands for estimated value based on the proposed equations. Density, water content at saturation and apparent porosity are also some of the important properties that influence the behaviour of rock (Goodman, 1993). So, the physical properties, which include dry density, saturated density, grain density, water content at saturation and apparent porosity, have also been assessed as extra technical information for better understanding of the assessed rocks. All the above said investigations have been done as per ISRM Suggested Methods (Blue Book, 2006).

II. DISCUSSION

In this paper, the discussion is done in five stages. First, the representative values of the above said properties and parameters have been discussed, to have an understanding about the three variants of the sandstone. Second, complete spread of the data of UCS, E, V_p and V_s is shown. Third, correspondence of these values with the equations proposed for sandstone by other researchers has been assessed. Forth, correlations between UCS and E with V_p and V_s have been explored through regression analysis. Fifth, the implication of using the proposed correlations has been assessed on the same data and the sandstone data (UCS and V_p) of Mishra and Basu, 2013. This assessment of implication is done by plotting of actual UCS (UCSa) and estimated UCS from the correlations (UCSe), percentage error in estimating UCS through the equations and variation in the estimated range of values to that of actual.

A. Representative Values

The representative values of the foresaid properties and parameters for the three sandstones are shown in Table 1. The representative value is based on the holistic analysis of the rock, i.e., truncating the non-representative values from the data of consideration. However, in the later stages of the paper, each and every value has been taken into consideration.

Table - 1
Representative Values of Properties and Parameters of Sandstones

S.No	Parameter		SS1	SS2	SS3
1	<i>Identification and Water-related Properties</i>				
	<i>Bulk Density (kg/m³)</i>	<i>Y_{dry}</i>	2490	2590	2648
		<i>Y_{sat}</i>	2600	2625	2658
	<i>Grain Density (kg/m³)</i>	<i>Y_{grain}</i>	2640	2640	2686
	<i>Water Content at sat. (%)</i>	<i>wc</i>	6.00	2.00	0.45
	<i>Apparent Porosity (%)</i>	<i>η</i>	15.00	5.00	1.19
2	<i>Wave Velocity (km/sec)</i>				
	<i>Compression Wave Velocity</i>	<i>V_p (dry)</i>	2.50	2.40	4.90
		<i>V_p (sat)</i>	2.80	2.50	5.00
	<i>Shear Wave Velocity</i>	<i>V_s (dry)</i>	1.20	1.80	2.50
		<i>V_s (sat)</i>	1.40	1.90	2.60
3	<i>Strength & Deformability Characteristics in Uniaxial Compression(saturated)</i>				
	<i>Uniaxial Comp. Strength, (MPa)</i>	<i>UCS</i>	13.00	10.00	100.00
	<i>Tangent Modulus, (GPa)</i>	<i>E</i>	9.00	---	40.00
	<i>Poisson's ratio</i>	<i>μ</i>	0.25	---	0.19

Comparing SS3 rock with SS1 & SS2, it is observed that SS3 possess high density whether it is Dry, Sat or Grain and having lowest water content due to low porosity as compared with the other two rocks. Test data shows that due to stronger molecular bonding and higher density SS3 possess higher UCS, E, V_p and V_s . Further, in respect of compressive strength, SS2 rock is slightly lower than SS1. However, SS2 is denser than SS1. But, water at content at saturation and apparent porosity of SS1 is higher than SS2.

Foregoing infers that, at finer scale (where variation is not considerable), there is no correlation between physical properties of rock and strength parameters. At finer scales, non-correlation between the two could be due to the shape factor; because while assessing physical properties sample taken is far smaller than that used in strength parameter which in turn reduces the amount of fissures present in the sample.

B. Data

UCS, E, V_p and V_s data of the three sand stones are presented in Figure 1, 2 and 3, respectively. Figure 1 infers that the UCS of SS1, SS2 and SS3 has 3, 4 and 3 fold variation. Figure 2 shows the variation of E of SS1 and SS3, which is 6 and 2 fold. Figure 3 shows the waves' velocities of SS1, SS2 and SS3, in both dry and saturated states. It can be inferred from Figure 3 that SS3 has higher waves' velocities than the other two, similar to UCS and E. Thill and Bur (1969) stated that the V_p changes with porosity and degree of saturation. Lama and Vutukuri (1978) indicated that the wetting of rock usually leads to a rise in V_p . Similarly, in this work also, both V_p and V_s are increasing on saturation.

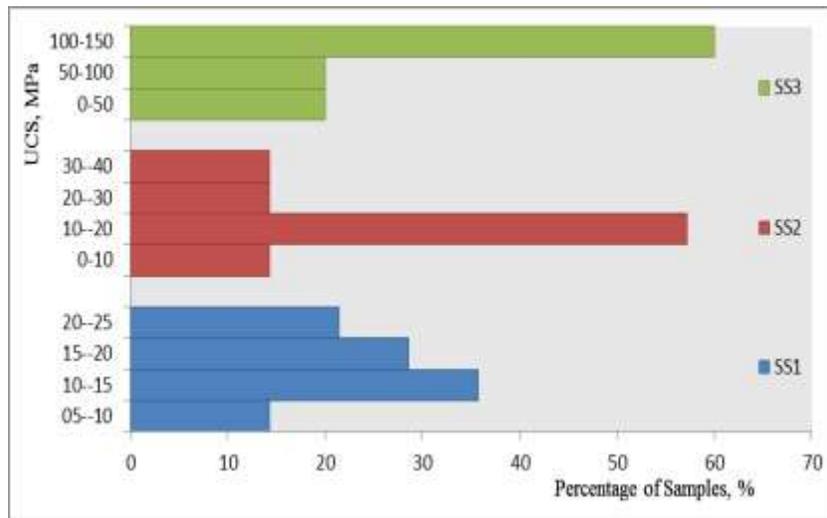


Fig. 1: UCS Data

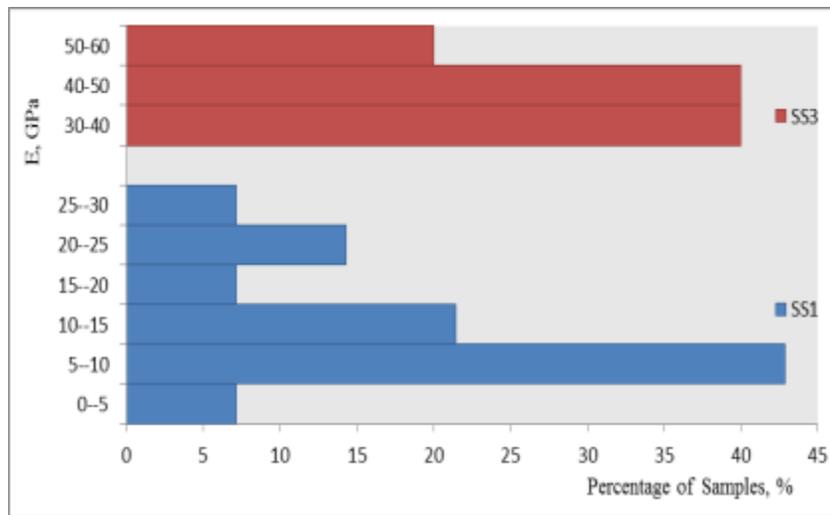


Fig. 2: E Data

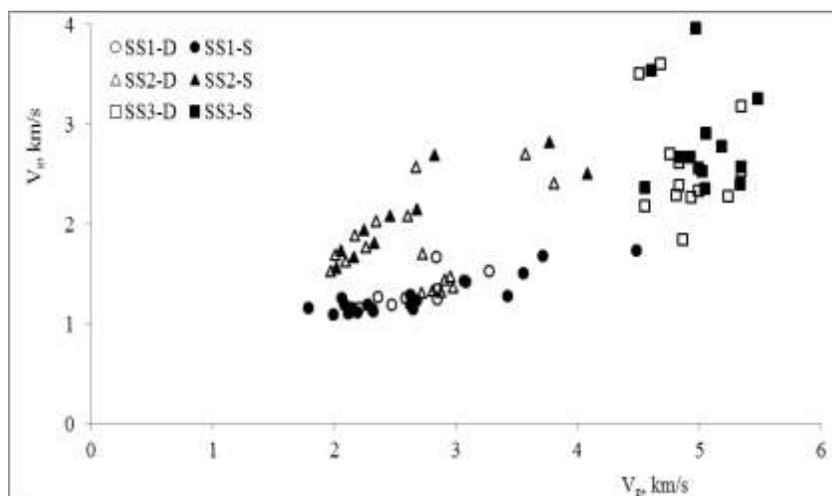


Fig. 3: Waves' Velocity Data (In both Dry and Saturated State)

C. Correlations for Sandstone Proposed by Other Researchers

Three researchers [Freyberg (1972); McNally (1987); two by Chary et al (2006)] have proposed four correlations for the assessments of UCS on the basis of V_p . Chary et al (2006) had proposed two correlations one for lower range and other for higher

range (i.e., below and above 4km/sec). Figure 4 shows the four correlation lines and the present data superimposed over it. It infers that the present data does not go with the correlations proposed by Freyberg (1972) and McNally (1987), while present data goes with Chary (2006). This means that V_p has a good correlation with UCS. But, in place of the two equations, it could be appropriate to give a single equation to avoid the confusion of using which of the two equations at confluence point (i.e., between 4 to 5km/sec.). However, it can easily be inferred that no correlation can be universal because the presented data is not fitting into the equations proposed by Freyberg and McNally and their data shall not fit into the equations of Chary or the equations proposed in this paper.

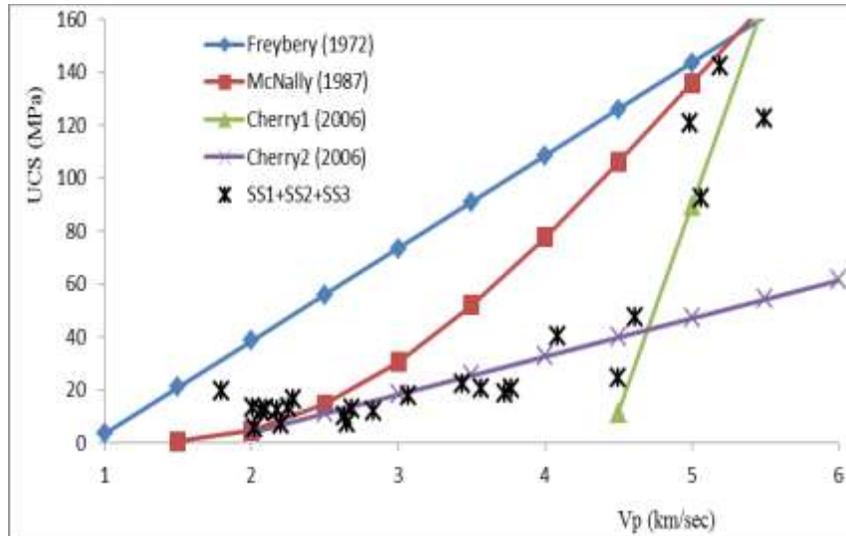


Fig. 4: UCS v/s V_p and Correlations of Sandstone by Other Researchers

D. Correlations

UCS and E have been correlated with V_p and V_s , both, using regression analysis. Figure 5 shows correlation between UCS and V_p , which gives a polynomial equation of third degree, with 0.90 as coefficient of regression. The equation is as under:

$$UCS = 3.65 V_p^3 - 22.37 V_p^2 + 40.77 V_p - 7.88 \quad (1)$$

Figure 6 shows correlation between UCS and V_s , which also gives a polynomial equation of third degree (ignoring four samples), with 0.95 as coefficient of regression. The equation is as under:

$$UCS = -17.05 V_s^3 + 134.07 V_s^2 - 278.38 V_s + 184.12 \quad (2)$$

Figure 7 shows correlation between E and V_p , which gives a polynomial equation of second degree (ignoring one sample), with 0.95 as coefficient of regression. The equation is as under:

$$E = 5.255 V_p^2 - 26.437 V_p + 40.9 \quad (3)$$

Figure 8 shows correlation between E and V_s , which gives a polynomial equation of second degree (ignoring 3 samples), with 0.96 as coefficient of regression. The equation is as under:

$$E = 2.073 V_s^2 + 13.687 V_s - 10.636 \quad (4)$$

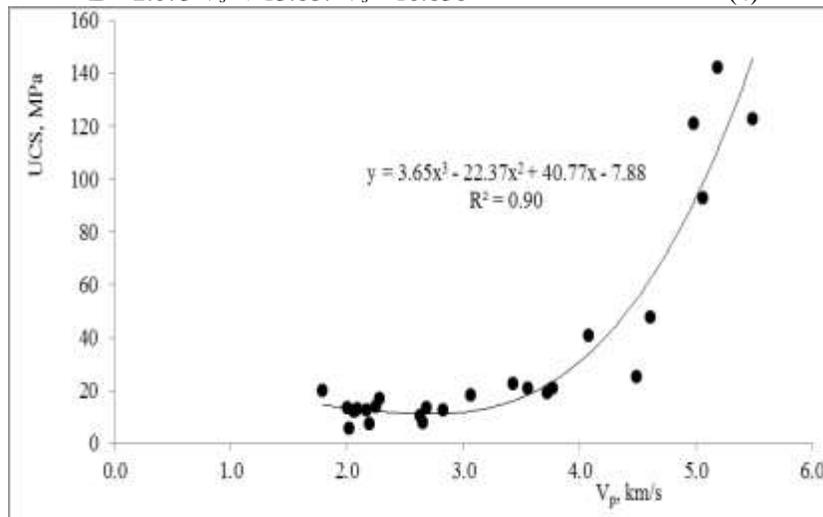


Fig. 5: Correlation between UCS and V_p

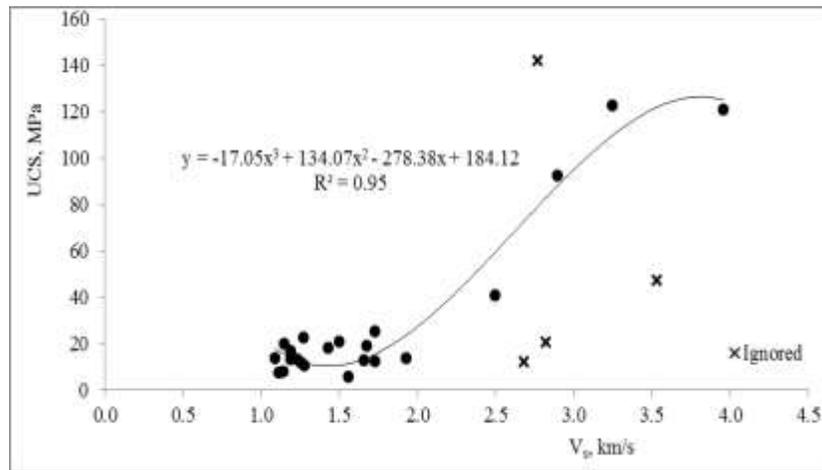


Fig. 6: Correlation between UCS and V_p

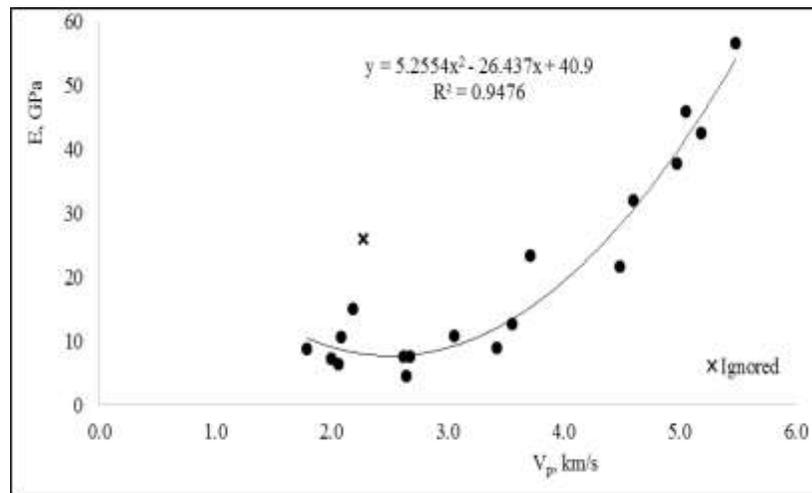


Fig. 7: Correlation between E and V_p

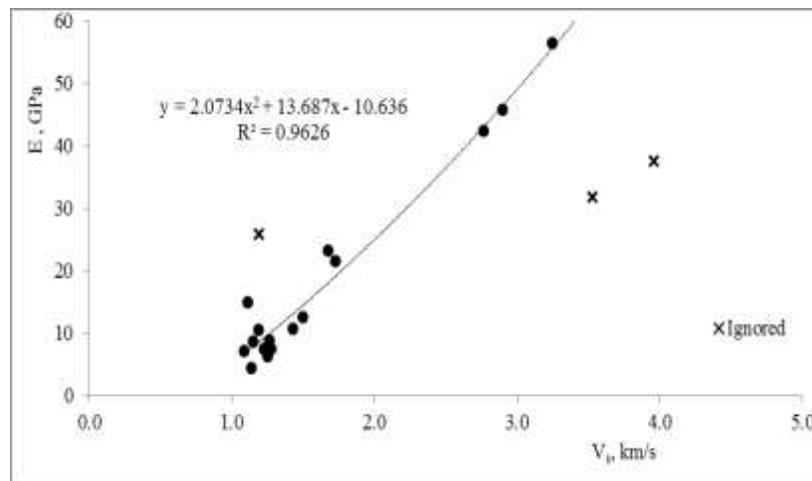


Fig. 8: Correlation between E and V_s

E. Implications of Using Equations

To assess the implications of using the above four equations, on the same data and on the data given by Mishra and Basu (2013) [only UCS and V_p is given], UCS and E have been estimated using the four equations. This assessment of implication has been done in three ways: (a) by plotting the estimated values against the actual values; (b) by calculating error in predicting the values; (c) by comparing the range provided by estimated values to that of actual values (reason for comparing the range is that rock being

DIANE, there cannot be any one to one relationship, but it could be fruitful, even if, one gets a broader picture of the rock, may be in the form of range).

1) Presented Data

The four equations have been derived from the presented data. So, to start with its implication of using the equations, it is thought to be appropriate to use the equations on its mother data and assess its applicability, before going forward to use universally. This assessment of each equation has been done in the three ways mentioned before.

a) Equation 1

In Figure 9, estimated values have been plotted with the actual values. It can be inferred from the Figure 9 that, by using Equation 1 (correlation between UCS and V_p), most of the estimated values are nearer to the actual values, but under estimated. Figure 10 shows the percentage error in estimating the UCS from V_p using Equation 1. It shows that around 55% estimated values are having an error of within $\pm 20\%$ and around 80% of the estimated values are within $\pm 40\%$. Figure 11 shows the actual and estimated ranges of the three sandstones. These actual and estimated ranges have been arrived by neglecting one or two values which had very wide difference from rest of the data. It can be inferred from the Figure 11 that, for SS1 and SS2, there is not much a difference in the ranges of actual and estimated data. While, in case of SS3, the range of estimated values are lesser than that of actual, but skewed towards lower side; which means that the estimated values providing a conservative value, which can also be inferred from Figure 9 and 10.

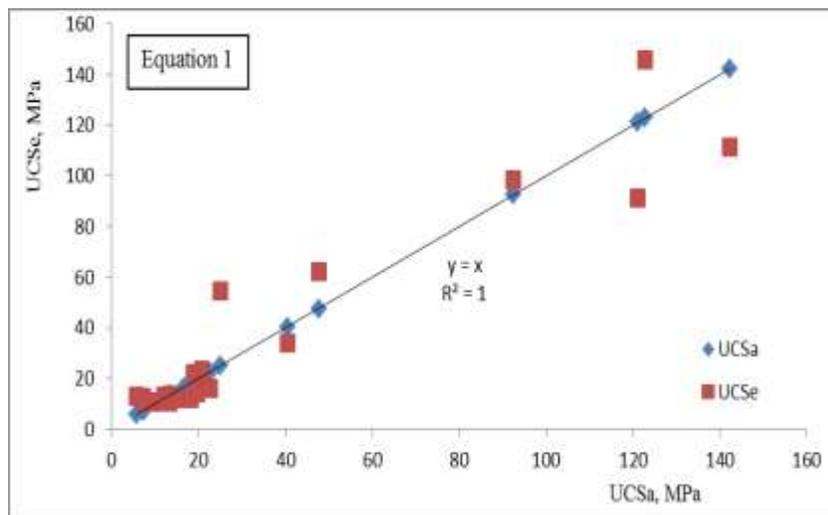


Fig. 9: UCS_e from Eq. 1 v/s UCS_a of Presented Data

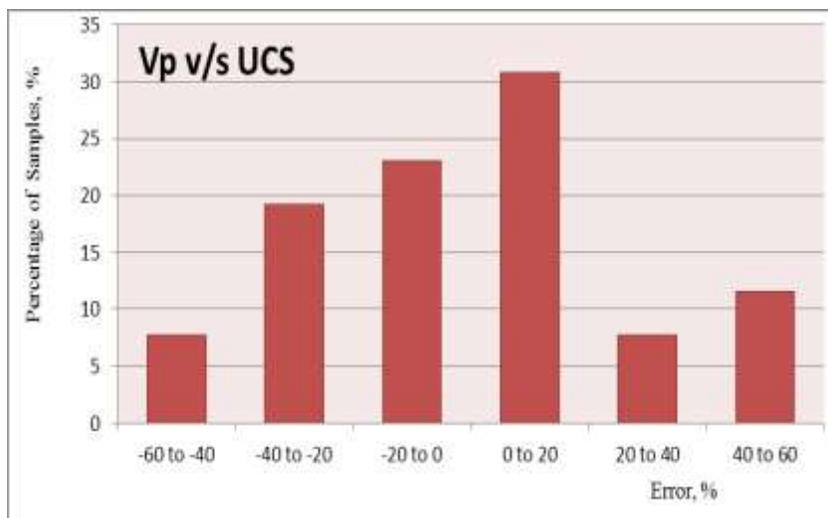


Fig. 10: Percentage Error in Estimation of Presented UCS from Eq. 1

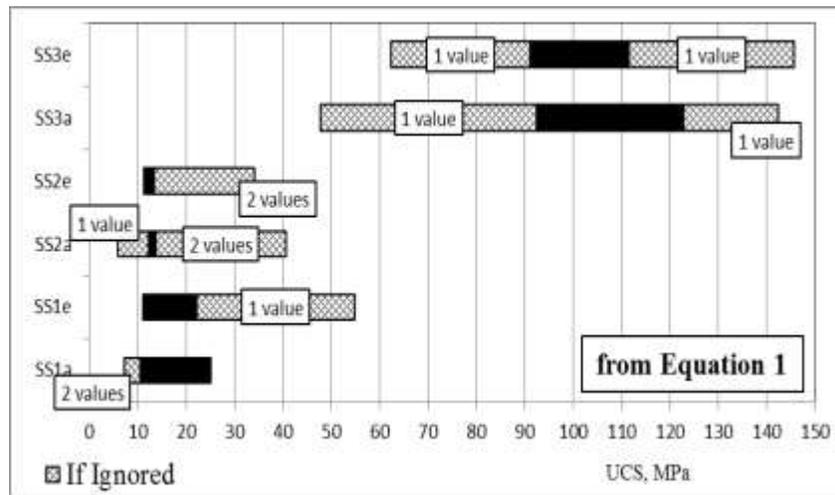


Fig. 11: Variation in the Range of UCS Given by Eq. 1 v/s Actual Presented Data

b) Equation 2

Estimated values obtained from Equation 2 (correlation between UCS and V_s) have been plotted with the actual values, in Figure 12. Figure 13 shows the percentage error in estimating the UCS from V_s using Equation-2. It shows that around 40% estimated values are having an error of within $\pm 20\%$ and around 60% of the estimated values are within $\pm 40\%$. And, rest around 40% estimated values are providing more than 40% error. Figure 14 shows the actual and estimated ranges of the three sandstones. These actual and estimated ranges have been arrived by neglecting one or two values which had very wide difference from rest of the data. It can be inferred from the Figure 14 that, for SS1, the range of estimated values are lesser than that of actual, but skewed towards lower side; which means that the estimated values providing a conservative value. While, in case of SS2 and SS3, the range of estimated values are widening in comparison to the actual range. The foregoing infers that the usage of Equation 2 shall provide a considerable error in estimation of UCS.

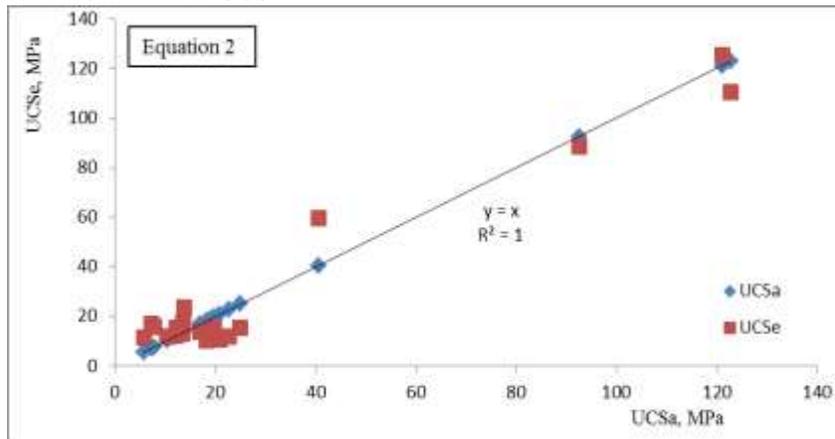


Fig. 12: UCS_e from Eq. 2 v/s UCS_a of Presented Data

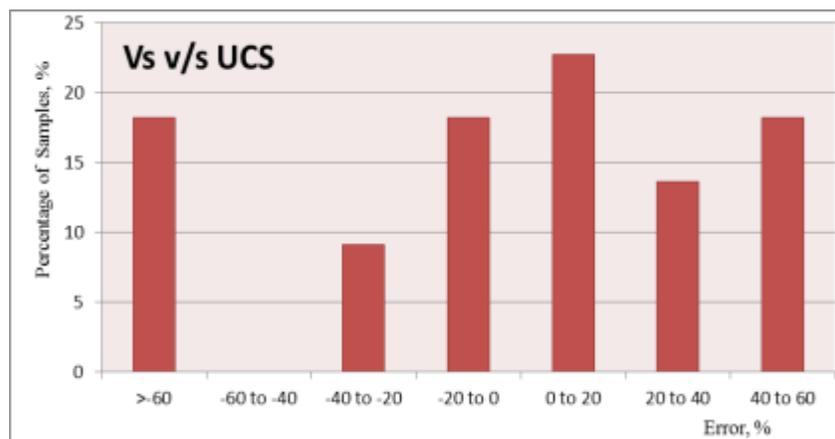


Fig. 13: Percentage Error in Estimation of Presented UCS from Eq. 2

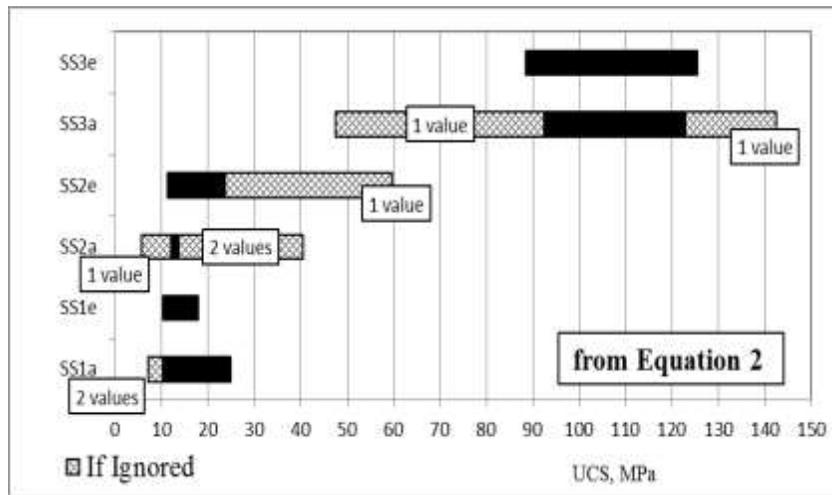


Fig. 14: Variation in the Range of UCS given by Eq. 2 v/s Actual Presented Data

c) Equation 3

Estimated values obtained from the second degree equation (Equation 3) between E and V_p have been plotted with the actual values, in Figure 15. Figure 15 infers that the most of the estimated are over estimated. Figure 16 shows the percentage error in estimating the E from V_p using Equation 3. It shows that around 45% estimated values are having an error of within $\pm 10\%$ and around 83% of the estimated values are within $\pm 30\%$. And, rest around 15% estimated values are providing more than 30% error. It is also noted that around 60% of the data is over estimated, which can be harmful/ misleading even in the initial stages of a project. Figure 17 shows the actual and estimated ranges of SS1 and SS3. In case of SS1, actual range was 5 to 15GPa, while the estimated range from Equation 3 is 7 to 15GPa. In case of SS3, actual range was 32 to 56GPa, while the estimated range from Equation 3 is 31 to 54GPa. These estimated ranges provided by the Equation 3 are justifiable; rest depends upon the sensitivity required in design and project. In all, if one is looking for one to one relationship, Equation 3 is over estimating, but it is providing a perfect range (atleast for the presented data).

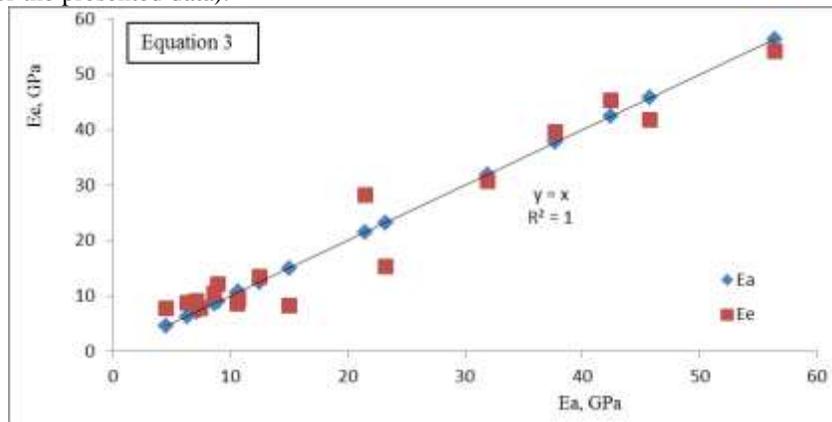


Fig. 15: E_e from Eq. 3 v/s E_a of Presented Data

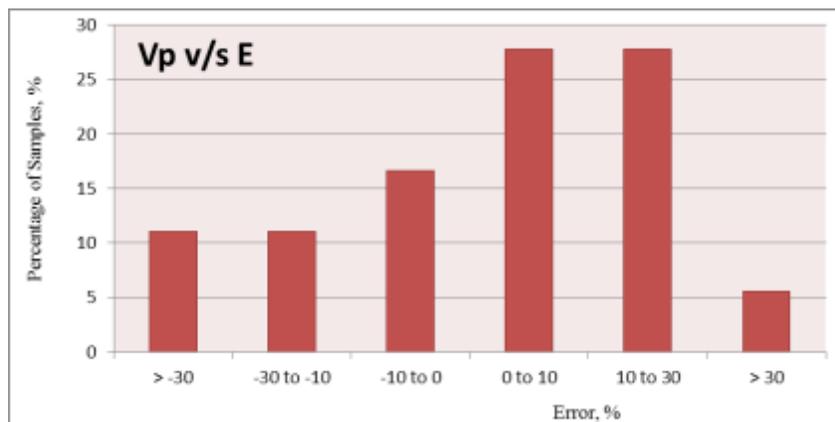


Fig. 16: Percentage Error in Estimation of Presented E from Eq. 3

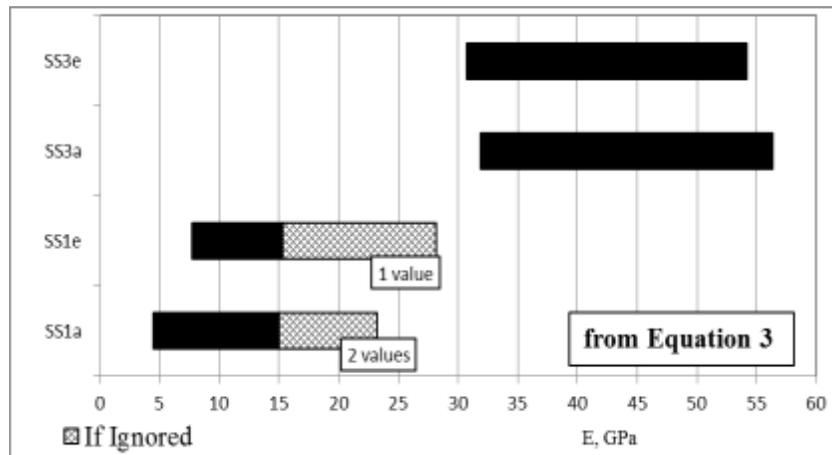


Fig. 17: Variation in the Range of E given by Eq. 3 v/s Actual Presented Data

d) Equation 4

Estimated values obtained from Equation 4 (correlation between E and V_s) have been plotted with the actual values, in Figure 18. It can be inferred from Figure 18 that most of the estimated values are nearer to the actual values, but over estimated. Figure 19 shows the percentage error in estimating the UCS from V_s using Equation 4. It shows that around 62% estimated values are having an error of within $\pm 20\%$ and around 90% of the estimated values are within $\pm 30\%$. Figure 20 shows the actual and estimated ranges of SS1 and SS3. For both, SS1 and SS3, estimated ranges are shrinking and skewed towards higher side, providing over estimated range (similar inference was drawn from Figure 18).

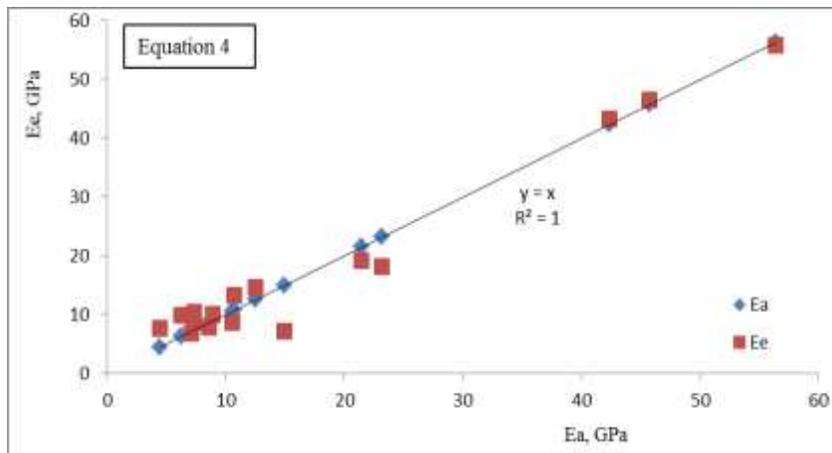


Fig. 18: E_e from Eq. 4 v/s E_a of Presented Data

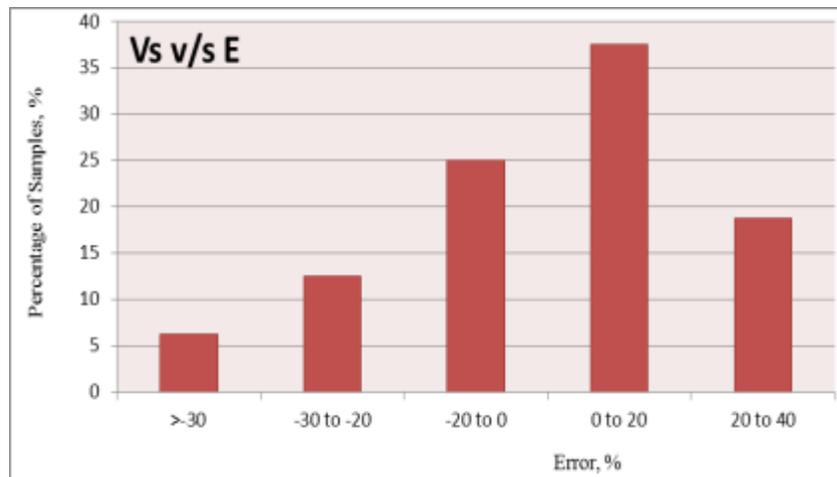


Fig. 19: Percentage Error in Estimation of Presented E from Eq. 4

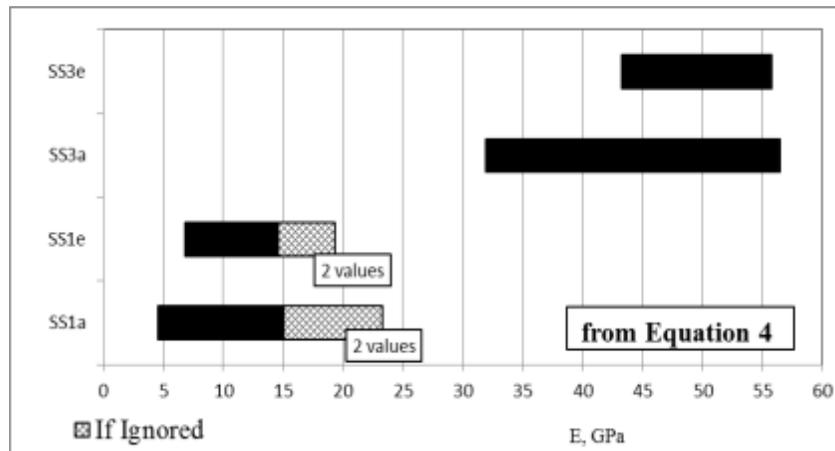


Fig. 20: Variation in the Range of E given by Eq. 4 v/s Actual Presented Data

2) Mishra and BASU (2013)

On the basis of V_p provided by Mishra and Basu (2013), UCS values has been estimated using Equation 1 and are compared with the actual UCS data provided there in. The estimated data and the actual data have been presented in Figure 21, which infers that the estimated values far lower than that of actual. Figure 22 shows the percentage error, which infers that 90% of the estimated values are having errors between -25 to -75%. Figure 23, showing ranges of actual and estimated UCS values, infers that the estimated range has shrunk and skewed towards lower side, i.e., lower range is approximately equal, while upper range is far lower than that of actual.

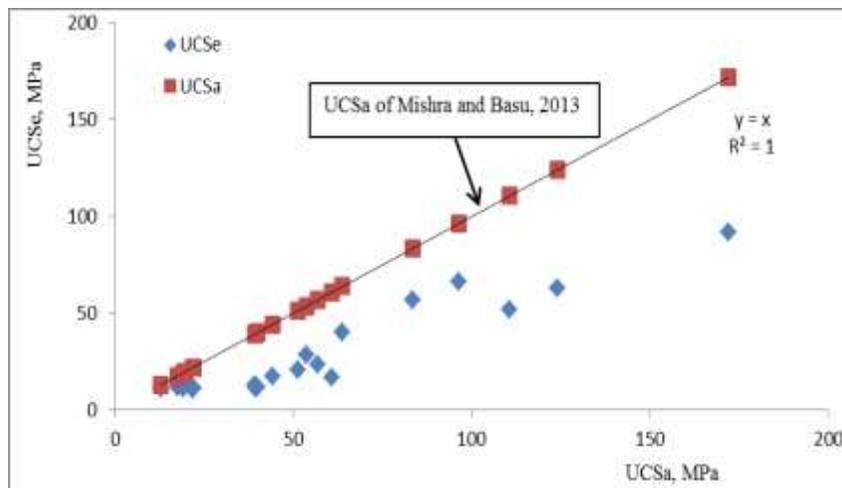


Fig. 21: UCS_e from Eq. 1 v/s UCS_a of Mishra and Basu, 2013

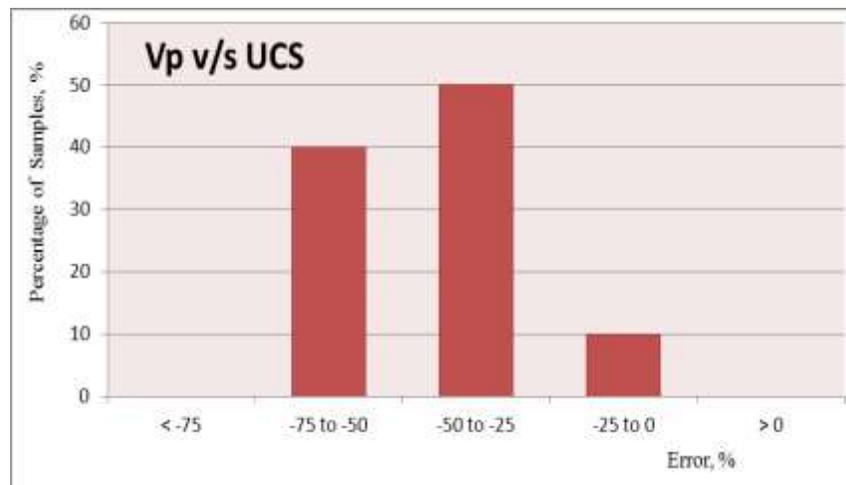


Fig. 22: Percentage Error in Estimation of E of Mishra and Basu, 2013 from Eq. 1

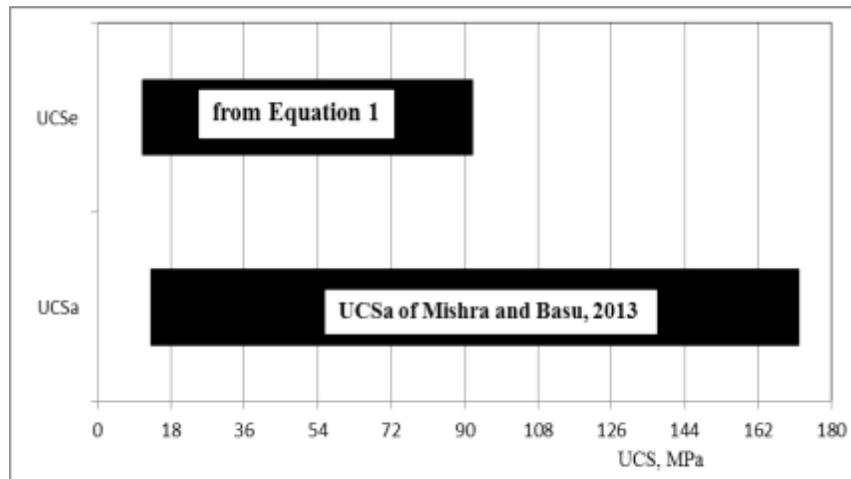


Fig. 23: Variation in the Range of UCS given by Eq. 1 v/s Actual Data of Mishra and Basu, 2013

III. CONCLUSIONS

From the above discussions, it can easily be concluded that, correlations are rock type and location specific; with percentage of errors ranging from 30 to 40. However, in preliminary stage of any project, a broad picture of the rock can be obtained using these equations, may be in the form of range. But, it is sure that these or any correlations can never be an alternative for the investigation.

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