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Relation between Fuzzy Subgroups & Anti-Fuzzy Subgroups

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Abstract

In this paper, we discuss the definitions of union of Fuzzy subsets, the definitions of union of Anti-Fuzzy subsets, definition of fuzzy subgroup, definition of anti- fuzzy subgroup, and show the relation between fuzzy subgroups and an Anti- fuzzy subgroups and derive some theorems.

Keywords: Fuzzy Subsets, Anti-Fuzzy Subsets, Compact, Closed Sets, Compositions

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I. Introduction

The concept of fuzzy sets was initiated by Zadeh. Then it has become a vigorous area of research in engineering, medical science, social science, graph theory etc. Since the paper fuzzy set theory has been considerably developed by zadeh himself and some researchers. The original concept of fuzzy sets was introduced as an extension of crisps (usual) sets, by enlarging the truth value set of "grade of members "from the two value set {0,1} to unit interval {0,1} of real numbers. The study of fuzzy group was started by Rosenfeld. It was extended by Roventa who have introduced the fuzzy groups operating on fuzzy sets.

II. Preliminaries

In this section contain some definitions, examples and some results.

A. *Concept of a Fuzzy Set*

The concept of a fuzzy set is an extension of the concept of a crisp set. Just as a crisp set on a universal set U is defined by its characteristic function from U to {0,1} , a fuzzy set on a domain U is defined by its membership function from U to [0,1] .

Let U be a non-empty set, to be called the Universal set (or) Universe of discourse or simply a domain.

Then, by a fuzzy set on U is meant a function $A: U \to [0, 1]$. A is called the membership function; A (x) is called the membership grade of x in A. We also write

$$A= \{(x, A (x)): x \in U\}.$$

1) *Examples*

Consider U={a ,b ,c ,d} and A:U→1 defined by A(a)=0,A(b)=0.7,A(c)=0.4,and A(d)=1.Then A is a fuzzy set can also be written as follows:

$$A= \{(a, 0), (b,0.7),(c,0.4),(d,1)\}.$$

B. *Relation between Fuzzy Sets*

Let U be a domain and A, B be fuzzy sets on U.

Inclusion (or) Containment: A is said to be included (or) contained in B if and only if $A(x) \leq B(x)$ for all x in U.

In symbols, we write, $A \subseteq B$. We also say that A is a subset of B.

C. *Definition*

Let S be a set. A fuzzy subset A of S is a function $A: S \to [0, 1]$.

D. *Definition of Union of Fuzzy sets:*

The union of two fuzzy subsets μ_1,μ_2 is defined by $(\mu_1 \cup \mu_2)(x) = \max\{\mu_1(x),\mu_2(x)\}$ for every x in U.

E. *Definition of Fuzzy Union of the fuzzy sets μ_1 and μ_2*

Let μ_1 be a fuzzy subset of a set x_1 and μ_2 be a fuzzy subset of a set x_2 , then the fuzzy union of the fuzzy sets μ_1 and μ_2 is defined as a function.

$\mu_1 \cup \mu_2 : x_1 \cup x_2 \rightarrow [0,1]$ given by

$$(\mu_1 \cup \mu_2)(x) = \begin{cases} \max(\mu_1(x), \mu_2(x)) & \text{if } x \in x_1 \cap x_2 \\ \mu_1(x) & \text{If } x \in x_1 \,\&\, x \notin x_2 \\ \mu_2(x) & \text{If } x \in x_2 \,\&\, x \notin x_1 \end{cases}$$

F. Definition of Union of Anti - Fuzzy Sets

The union of two fuzzy subsets A_1, A_2 is defined by $(A_1 \cup A_2)(x) = \min\{A_1(x), A_2(x)\}$ for every x in U.

G. Definition of Anti- Fuzzy union of the fuzzy sets A_1 and A_2

Let A_1 be a fuzzy subset of a set x_1 and A_2 be a fuzzy subset of a set x_2 , then the anti-fuzzy union of the fuzzy sets A_1 and A_2 is defined as a function.
$A_1 \cup A_2 : x_1 \cup x_2 \rightarrow [0,1]$ given by

$$(A_1 \cup A_2)(x) = \begin{cases} \min(A_1(x), A_2(x)) & \text{if } x \in x_1 \cap x_2 \\ A_1(x) & \text{if } x \in x_1 \,\&\, x \notin x_2 \\ A_2(x) & \text{if } x \in x_2 \,\&\, x \notin x_1 \end{cases}$$

H. Definition of Fuzzy Subgroup

Let G be a group. A fuzzy subset μ of a group G is called a fuzzy subgroup of the group G if
1) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$ for every $x, y \in G$. And
2) $\mu(x^{-1}) = \mu(x)$ for every $x \in G$.

I. Definition of Anti Fuzzy Subgroup

Let G be a group. A fuzzy subset μof a group G is called an anti-fuzzy subgroup of the group G if
1) $\mu(xy) \leq \max\{\mu(x), \mu(y)\}$ for every $x, y \in G$ and
2) $\mu(x^{-1}) = \mu(x)$ for every $x \in G$.

From the definitions 2.8. And 2.9. We have min

$$\{\mu(x), \mu(y)\} \leq \mu(xy) \leq \max\{\mu(x), \mu(y)\} \quad (1)$$

III. THEOREMS

A. Theorem 3.1

By condition 1, prove that μ is compact.

1) Proof

w.k.t
By Hein –Borel Theorem "The set is compact if and only if it is closed and bounded".
It is enough to prove that **μ** is closed and bounded.
The equation 1 is compared with

$$a \leq x \leq b, \text{ it is closed.}$$

Obviously, μ is closed,
By 1

$$\mu(xy) \leq \max\{\mu(x), \mu(y)\} = M,$$

Obviously μ is bounded.
Thus μ is closed and bounded.
Hence μ is compact.

B. Theorem 3.2

A Closed subset of a compact set is compact.

1) Proof

Let μ_1 and μ_2 are subsets of a compact set,
$\mu_1 \subseteq \mu$ And $\mu_2 \subseteq \mu$
By 1,

$$\text{Min}\{\mu(x),\mu(y)\} \leq \mu_1(xy) \leq \mu(xy) \leq \max\{\mu(x),\mu(y)\} \quad (2)$$
$$\text{Min}\{\mu(x),\mu(y)\} \leq \mu_2(xy) \leq \mu(xy) \leq \max\{\mu(x),\mu(y)\} \quad (3)$$

From 2 and 3, Thus μ_1 and μ_2 are closed and bounded.
By Theorem 1,
Hence μ_1 and μ_2 are compact.

C. *Theorem 3.3*

A Finite union of compact sets is compact.
1) *Proof*
Let μ_1 and μ_2 are compact subsets of a compact set ,

$$\mu_1 \subseteq \mu_1 \cup \mu_2 \subseteq \mu \text{ And } \mu_2 \subseteq \mu_1 \cup \mu_2 \subseteq \mu .$$

by theorem 2 ,
Thus $\mu_1 \cup \mu_2$ is closed and bounded.
Thus $\mu_1 \cup \mu_2$ is compact.
Hence a Finite union of compact sets is compact.

D. *Theorem 3.4*

The Product of any collection of compact sets is compact.
1) *Proof*
Let μ_1 and μ_2 are compact subsets of a compact set ,
By above theorems,
$\mu_1 \circ \mu_2$ is also compact .

E. *Theorem 3.5:*

If $\mu_1 \circ \mu_2$ is a fuzzy subgroup of a group G if and only if $\mu_1 \circ \mu_2 = \mu_2 \circ \mu_1$.
1) *Proof*
a) Necessary Part

$$\text{Let } \mu_1 \circ \mu_2 = \mu_2 \circ \mu_1$$

To prove:
$\mu_1 \circ \mu_2$ is a fuzzy subgroup of a group G .
Let $\mu_1$and μ_2 are fuzzy subgroup of a group G
$\mu_1 \circ \mu_2(xy) = \mu_1(\mu_2(xy))$, where x, y ϵG
Since μ_2 is a fuzzy subgroup of a group G.
by the definition of fuzzy subgroup ,
1) $\mu_2(xy) \geq \min\{\mu_2(x),\mu_2(y)\}$for every x,y∈G.

$$\mu_1 \circ \mu_2(xy) \geq \mu_1(\min\{\mu_2(x), \mu_2(y)\})$$
$$\geq \min \mu_1(\{\mu_2(x), \mu_2(y)\})$$
$$\mu_1 \circ \mu_2(xy) \geq \min\{\mu_1 \circ \mu_2(x), \mu_1 \circ \mu_2(y)\}) \quad (1)$$
$$\mu_1 \circ \mu_2(x^{-1}) = \mu_1(\mu_2(x^{-1}))$$

Again by the definition of fuzzy subgroup,

$$\mu_2(x^{-1}) = \mu_2(x) \text{ for every } x \in G.$$
$$\mu_1 \circ \mu_2(x^{-1}) = \mu_1(\mu_2(x))$$
$$\mu_1 \circ \mu_2(x^{-1}) = \mu_1 \circ \mu_2(x) \quad (2)$$

From 1 and 2,
$\mu_1 \circ \mu_2$ is a fuzzy subgroup of a group G .
b) Sufficient Part
Let $\mu_1 \circ \mu_2$ is a fuzzy subgroup of a group G.
To prove:

$$\mu_1 \circ \mu_2 = \mu_2 \circ \mu_1$$

by 1 ,

$$\mu_1 \circ \mu_2(xy) \geq \mu_2(min\{\mu_1(x), \mu_1(y)\})$$

Since by the definition of fuzzy subgroup,
2) $\mu_1(xy) \geq \min\{\mu_1(x),\mu_1(y)\}$for every x,y∈G.

$$\mu_1 \circ \mu_2(xy) \geq \mu_2(\mu_1(xy))$$
$$\mu_1 \circ \mu_2(xy) \geq \mu_2 \circ \mu_1(xy).$$
$$\mu_1 \circ \mu_2 \supseteq \mu_2 \circ \mu_1.$$
$$\mu_2 \circ \mu_1 \subseteq \mu_1 \circ \mu_2 \quad (3)$$

Similarly, we get,

$$\mu_1 \circ \mu_2 \subseteq \mu_2 \circ \mu_1 \quad (4)$$

From 3 and 4,

$$\mu_1 \circ \mu_2 = \mu_2 \circ \mu_1 .$$

Hence,
If $\mu_1 \circ \mu_2$ is a fuzzy subgroup of a group G if and only if $\mu_1 \circ \mu_2 = \mu_2 \circ \mu_1$.
Thus proved.
For an Anti-fuzzy subgroups, we use A instead of μ.

F. Theorem 3.5

If $A_1 \circ A_2$ is an Anti- fuzzy subgroup of a group G if and only if $A_1 \circ A_2 = A_2 \circ A_1$.
1) Proof
a) Necessary part

$$\text{Let } A_1 \circ A_2 = A_2 \circ A_1$$

To prove:
$A_1 \circ A_2$ is an Anti-fuzzy subgroup of a group G .
Let $A_1 and\ A_2$ are an Anti- fuzzy subgroup of a group G
$A_1 \circ A_2(xy) = A_1(A_2(xy))$, where x,y ϵG
Since A_2 is an Anti- fuzzy subgroup of a group G.
by the definition of an Anti-fuzzy subgroup ,
3) $A_2(\text{xy}) \leq \max \{A_2(\text{x}), A_2(\text{y})\}$ for every x,y∈G.
$A_1 \circ A_2(xy) \leq A_1(\max \{A_2(\text{x}), A_2(\text{y})\})$
$\leq \max A_1(\{A_2(\text{x}), A_2(\text{y})\})$

$$A_1 \circ A_2(xy) \leq \max\{ A_1 \circ A_2(\text{x}),\ A_1 \circ A_2(\text{y})\}) \quad (1)$$

$A_1 \circ A_2(\text{x}^{-1}) = A_1(A_2(\text{x}^{-1}))$
Again by the definition of an Anti-fuzzy subgroup,
$A_2(\text{x}^{-1}) = A_2(\text{x})$ for every x∈G.
$A_1 \circ A_2(\text{x}^{-1}) = A_1(A_2(\text{x}))$

$$A_1 \circ A_2(\text{x}^{-1}) = A_1 \circ A_2(\text{x}) \quad (2)$$

From 1 and 2,
$A_1 \circ A_2$ is an Anti- fuzzy subgroup of a group G .
b) Sufficient Part
Let $A_1 \circ A_2$ is an Anti- fuzzy subgroup of a group G.
To prove

$$A_1 \circ A_2 = A_2 \circ A_1$$

by 1 ,
$A_1 \circ A_2(xy) \leq A_2(max\{A_1(\text{x}), A_1(\text{y})\})$
Since by the definition of an Anti- fuzzy subgroup,
4) $A_1(\text{xy}) \leq \max \{A_1(\text{x}), A_1(\text{y})\}$ for every x,y∈G.

$$A_1 \circ A_2(xy) \leq A_2(A_1(\text{xy}))$$
$$A_1 \circ A_2(xy) \leq A_2 \circ A_1(xy)$$
$$A_2 \circ A_1 \subseteq A_1 \circ A_2 \quad (3)$$

Similarly,
We get,

$$A_1 \circ A_2 \subseteq A_2 \circ A_1 \quad (4)$$

From 3 and 4,

$$A_1 \circ A_2 = A_2 \circ A_1$$

Hence,
If $A_1 \circ A_2$ is a fuzzy subgroup of a group G if and only if $A_1 \circ A_2 = A_2 \circ A_1$.
Thus proved.

IV. CONCLUSION

We derive the Relation between Fuzzy subgroups and Anti-Fuzzy subgroups and shows that the set is compact and their union is also compact, compact properties and shown the composition properties.

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